CS2022/MA2201
HW#6 SOLUTIONS

1) Let $r : Z \rightarrow \{0,\ldots,d-1\}$ denote the remainder upon division by d, that is $r(n)=n \bmod d$. Since the cardinality of the codomain is d, by the Pigeonhole Principle for any $d+1$ elemernts in the domain must send at least two of these elements to the same image in the codomain.

2) ***a***) 5 ***b***) 10 ***c***) 70 ***d***) 1 ***e***) 1 ***f***) 824

3) $C(m+n,r)$ and $C(36,42)=0$

4) $C(40,17)=C(40,23)$ which is the number of ways to choose which questions are true which is the number of ways to choose which questions are false.

5) Jill gets more heads if any of the following three events occurs:
- Jill gets 2 heads & Jack gets 0 heads
- Jill gets 1 head and Jack gets 0 heads
- Jill gets 2 heads and Jack gets 1 head

The three mutually exclusive events have probabilities $(1/2)^3$, $(1/2)^2$ and $(1/2)^3$. The probability of Jill getting more heads is the sum of these probabilities, or 1/2 .

6) The probability of any sequence of three distinct digits being drawn is (1/10)*(1/9)*(1/8)=1/720, so all such sequences are equiprobable. Therefore, the experiment is equivalent to asking the probability that a random permutation (drawn from a uniform distribution) of {1,2,3} will be increasing. Each of the six permutations being equiprobable, the answer is 1/6.

7) The probability that D_i will beat D_j is the entry of row i and column j of the array

$$\begin{bmatrix} - & \frac{22}{36} & \frac{18}{36} & \frac{12}{36} \\ \frac{12}{36} & - & \frac{22}{36} & \frac{16}{36} \\ \frac{18}{36} & \frac{12}{36} & - & \frac{22}{36} \\ \frac{22}{36} & \frac{20}{36} & \frac{12}{36} & - \end{bmatrix}$$

For any choice of die that you make (say D_i), your opponent picks a die (say D_j) such that the corresponding entry in the array (the probability of you winning) is <1/2. Since every row has an entry 12/36, you lose.