CS2022/MA2201
HW#6 SOLUTIONS

1. If n=0, then there are no strings. If n=1, then there is one string, namely "1". If $n \geq 2$, then there are n-2 choices for the intermediate bits. Hence, the number of strings is 2^{n-2}.

2. (*a*) 14 (*b*) 27

3. 38+23-7=54

4. (*a*) The two colors are the pigeonholes. 4 balls won't suffice (she might have 2 red and 2 blue). She needs 5 balls since $\lceil 5/2 \rceil = 3$.
(*b*) She needs 13 balls. With fewer than 13 balls she might have up to 10 red balls and the rest blue.

5. We have 6 pigeons (the computers) and 5 pigeonholes labelled 1, 2, 3, 4 and 5. Each pigeon is put into the hole corresponding to the number of other computers to which it is connected. The Pigeonhole Principle assures that at one pigeonhole must receive at least two pigeons.

6. (*a*) $P(6,6) = 6! = 720$
(*b*) Treating (Boston, Providence) as a pair, $P(5,5) = 120$

7. (*a*) The committee must contain exactly three men and exactly two women. By the rule of product, this is the number of ways to choose the men times the number of ways to choose the women. This is $C(5,3) * C(7,2) = \frac{5!}{3!2!}\frac{7!}{2!5!} = 210$.
(*b*) In order to choose at least one man, we must choose one or two or three or four or five men (along with the appropriate number of women). We use the addition rule to add these cases, yielding

$$C(5,1)*C(7,4)+C(5,2)*C(7,3)+C(5,3)*C(7,2)+C(5,4)*C(7,1)+C(5,5)*C(7,0)$$

$$= \sum_{k=1}^{5} C(5,k)*C(7,5-k) = 771$$

(*c*) Using the same reasoning as in the last subproblem,
$C(5,0)*C(7,5)+C(5,1)*C(7,4)=196$