CS2022/MA2201
HW#6 SOLUTIONS

1. There are 4^{10} possible answer sheets, so $2*4^{10}+1$ students in the class would assure that at least three people had the same answer sheet.

2. ***a*** Let E_k, $1 \le k \le 100$, denote the event that you all pick *k*. For each *k*, the probability that any fixed person picks *k* is $\frac{1}{100} = 10^{-2}$, and $p(E_k) = (10^{-2})^{-100} = 10^{-200}$. Since $j \ne k \to E_j \cap E_k = \varnothing$,

$$p\left(\bigcup_{1\le k\le 100} E_k\right) = \sum_{1\le k\le 100} p(E_k) = \sum_{1\le k\le 100} 10^{-200} = 100*10^{-200} = 10^{-198}.$$

b For the binomial distribution, we label success as picking 42 (success has probability 10^{-2}), and failure as picking some number other than 42 (failure has probability $\frac{99}{100}$).
Thw probability of exactly 95 successes is

$$\binom{100}{95}\left(10^{-2}\right)^{95}\left(\frac{99}{100}\right)^{5} = .7159768240 \ \ 10^{-182}$$

c For the binomial distribution, we label success as picking a number greater than 40 (success has probability 0.6), and failure as picking some number ≤ 40 (failure has probability 0.4). The probability of at least 50 successes is

$$\sum_{50\le k\le 100}\binom{100}{k}(.6)^k(.4)^{100-k} = .9832383135$$

3. Consider the three events E_X, E_Y and E_Z, meaning that prisoner *X* (respectively *Y* and *Z*) survives. Since exactly one of E_X, E_Y and E_Z occurs, $E_X \cup E_Y \cup E_Z = \Omega$, and random (uniform) selection of a prisoner to survive means that $p(E_X) = p(E_Y) = p(E_Z) = \frac{1}{3}$.
Learning that prisoner *Y* dies (doesn't survive) means that event $\overline{E_Y}$ occurs. *X*'s new probability of survival is

$$p\left(E_X \mid \overline{E_Y}\right) = \frac{p\left(E_X \cap \overline{E_Y}\right)}{p\left(\overline{E_Y}\right)} = \frac{1/3}{2/3} = \frac{1}{2}$$

so his joy is justified.