CS2022/MA2201
HW#6

DUE: Monday, April 16

1. (8 points) Consider the set of all functions $f:\{1,...,n\}\to\{0,1\}$ where $n\in\mathbb{Z}^+$.
a How many such functions are there?
b How many such one-to-one functions are there?
c How many such onto functions are there?
d How many such bijections are there?

2. (8 points) For $a,b\in\mathbb{Z}$ we say that $a\,\mathrm{mod}\,b$ is the remainder upon dividing *a* by *b*. For example, $13\,\mathrm{mod}\,7=6$ and 14mod7=0 and for any $a,b\in\mathbb{Z}, a\,\mathrm{mod}\,b\in\{0,...,b-1\}$. How many ordered pairs of integers (a,b) are needed to guarantee that there are two ordered pairs (a_1,b_1) and (a_2,b_2) such that $a_1\,\mathrm{mod}\,5=a_2\,\mathrm{mod}\,5$ and $b_1\,\mathrm{mod}\,5=b_2\,\mathrm{mod}\,5$. Justify your response.

3. (8 points) In a computer network, each connection runs directly between a pair of computers, and such a pair of computers is called *directly connected*. If the network has *m* computers, then each computer is directly connected to $0,1,...,m-1$ other computers. Show that in **any** network there must exist at least two computers which are directly connected to the same number of other computers. Hint: Draw some networks and label each computer with the number of other computers to which it is directly connected.

4. (5 points) How many permutations of the letters G E O R W B U S H contain the (contiguous) substring R W B? Note that O E R G W B U S H does **not** contain the substring.

5. (8 points) ***a*** Suppose that 12 people are playing soccer. In how many ways (give a number as an answer) can the players be divided into two teams of 6 players each?
b Suppose that Gaddy and Al are among 12 people playing soccer. In how many ways (give a number as an answer) can the players be divided into two teams of 6 players each such that Gaddy and Al play on different teams?

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HW#6 SOLUTIONS

1. ***a*** 2^n
b For n=1 or n=2 there are two, and by the Pigeonhole Principle there are none for n>2.
c For n=1 there are 0 onto functions, and for n>1, of the 2^n functions, 2 of them are not onto. So for n>1 there are $2^n - 2$ onto functions.
d For n=2 there are 2 bijections, and for $n \neq 2$ there are 0 bijections.

2.We see from the product rule that there are 5*5=25 boxes into which we can place the objects, where we can label the boxes $\{0,1,2,3,4\} \times \{0,1,2,3,4\}$. So by the Pigeonhole Principle we need 26 objects to guarantee that 2 objects go into he same box.

3. Each of the m computers can be connected to $0,1,...,m-1$ other computers. Let these be the names of boxes (pigeonholes). The only way to place m objects (computers) into these boxes is to have exactly one object in each box. But in this case there is an object (computer) directly connected to 0 other computers and an object (computer) direcly connected to all of the m-1 other computers. This is impossible. So by contradiction there must exist two computers directly connected to exactly the same number of other computers.

4. If the permutation contains the substring R W B, then there are 7 places where it can start. For each of the 7 places, there are $P(6,6) = 6! = 720$ ways to permute the 6 other letters. By the sum rule, the sum of the cardinalities of these 7 subsets is $7 * P(6,6) = 7! = 5040$ ways to solve the problem.

5. ***a*** $\frac{C(12,6)}{2} = \frac{1}{2}\frac{12!}{6!6!} = 462$
b The number of ways to pick Al's team mates is $C(10,5) = 252$.