CS2022/MA2201
HW#6

DUE: Friday, October 3 (Covers material through the end of Chapter 5.)
For these problems, you needn't evaluate binomial coefficients. That is, you may leave terms like $C(852,255)$ in your answer.

1. (4 points) What is the probability of winning a lottery by correctly guessing all of 6 numbers which are chosen randomly from $\{1,...45\}$, where order doesn't matter? If you buy a lottery ticket for $1 and you win $1,000,000 by choosing a winning ticket, what is the value of a ticket after you buy it? That is, what are the expected winnings of the ticket?

2. (10 points) Do **EXERCISE 5.2-8** from our text.

3. (3 points) Assuming that everybody is equally likely to be born on any day of the week, and that these probabilities are independent, what is the probability that among n people, at least two were born on the same day of the week, for all $n \geq 1$?

4. (3 points) Assume that a factory produces chips, and the probability that a chip is defective is 0.003 Further assume that the probabilities that successive chips are defective are independent. What is the probability that in a batch of 1000 chips, at least 3 chips are defective?

5. (3 points) In throwing two fair dice, let random variable X count the number of dice that come up 6, and let Y be the sum of the two faces. Are X and Y independent? Justify your answer.

6. (3 points) Do **SUPPLEMENTARY EXERCISE 16** from pg. 396 our text.

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HW#6 SOLUTIONS

1. There are $C(45,6)$ ways to choose 6 numbers from 45, and the sample space *S* satisfies $|S| = C(45,6)$. Since the event of winning, $E_{winning}$, satisfies $|E_{winning}| = 1$, the probability of winning is $\frac{|E_{winning}|}{|S|} = \frac{1}{C(45,6)}$. By the way, Maple reveals that

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> binomial(45,6);
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8145060

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> evalf(1/binomial(45,6));
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0.1227738040 10^{-6}

Letting random variable *X* denote the winnings, the expected winnings are $E[X] = 1{,}000{,}000 * \Pr(X = 1{,}000{,}000) + 0 * \Pr(X = 0) = \$.122773$, yielding a 12¢ return on every 1$.

2. (a) ½

(b) ½

(c) There are $P(n,n) = n!$ permutations of *n*. Fixing 1 to immediately precede 2, we treat them as an entry and count the number of permutations of $\{12,3,4,\ldots,n\}$, and there are $P(n-1,n-1) = (n-1)!$ ways to do this. So the probability is $\frac{(n-1)!}{n!} = \frac{1}{n}$.

(d) In half the permutations *n* precedes 1 and in half the permutations *n*-1 precedes 2. Since these probabilities are independent (note that $n \geq 4$), we can multiply the probabilities to yield ¼.

(e) We can ignore the placements of 3,4,...,*n*-1, and then note that in 1/3 of the permutations of 1, 2 and *n* we have *n* preceding 1 **and** 2.

3. There are 7^n ways to assign days of the week (for birthdays) to *n* people, and $6*5*\ldots*(8-n)$ (where $6*5*\ldots*(8-n)$ =7 if *n*=1) ways to assign **different** birthdays to *n* people. Hence, the probability that the *n* people are all born on different days of the week is

$\frac{6*5*\ldots*(8-n)}{7^n}$, and the probability that two of them were born on the same day is

$1-\frac{6*5*\ldots*(8-n)}{7^n}$.

4. $\sum_{3\le i\le 1000} C(1000,i)(.003)^i(.997)^{1000-i} = 1-\sum_{0\le i\le 2} C(1000,i)(.003)^i(.997)^{1000-i}$

and Maple reveals that this probability is

0.5771465149

To appreciate the truly insignificant contribution of extreme end of the tail of this distribution, the probability that 1000 chips are all defective is

$$C(1000,1000)(.003)^{1000}(.007)^0 = (.003)^{1000} = 0.1322070819\ 10^{-2522}$$

5. X and Y are not independent.

$\Pr(X(s)=2 \wedge Y(s)=3) = 0 \ne \Pr(X(s)=2) * \Pr(Y(s)=3) = \frac{1}{36}\frac{2}{36} = .001543209877$

Intuitively, if you know that you threw a pair of 6s, this should strongly influence your knowledge of the sum of the faces of the dice.

6. If n is odd, this is impossible so the probability is 0. If n is even, then there are 2^n ways to flip a coin n times, and for $C(n,n/2)$ of these ways there are equal numbers of HEADs and TAILs. Hence the probability is $C(n,n/2)/2^n$.