

CS2022/MA2201
HW#6

DUE: Friday, October 5

1. (3 points) A professor teaching a Discrete Math course gives a multiple choice quiz that has 10 questions, each with 4 possible responses: a, b, c, d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? Assume that no answers are left blank.

2. (12 points) You and your 99 closest friends (there are a total of 100 people) each pick a number independently from a uniform distribution over the set $\{1, 2, \dots, 100\}$.

a What is the probability that you all pick identical numbers? That is, what is the probability that “you all pick 1 or you all pick 2 or...or you all pick 100”?

b What is the probability that exactly 95 of you pick the number 42?

c What is the probability that at least 50 of you pick a number greater than 40?

This answer needn't be in closed form.

3. (9 points) (From *Algorithms* by Cormen, Leiserson, Rivest & Stein)

A warden has randomly picked one prisoner among three prisoners $\{X, Y, Z\}$ to go free. The other two will be executed. The guard knows which one will go free but is forbidden to give any prisoner information about his status. X asks the guard privately to name one of Y and Z who will be executed, arguing that since he already knows that at least one of Y and Z will be executed, the guard won't be revealing anything about X 's status. The guard tells X that Y will be executed. X is now happier since he figures that one of X and Z will go free, which increases his probability of going free to $\frac{1}{2}$. With his new knowledge, is X right to be happier, or are his chances of survival still $\frac{1}{3}$? Give a careful probabilistic argument to explain.