CS2022/MA2201
HW#5 Solutions

1. Basis: $2^1 \leq 2*1!$
Induction Hypothesis: Assume that for some $n \geq 1$, $2^n \leq 2n!$. Then $2^{n+1} = 2 * 2^n$ and by the induction hypothesis $2*2^n \leq 2*2n! \leq (n+1)*2n! = 2(n+1)!$.

2. The proof is by induction on n. Basis: Since any three lines in general position can not meet in one point, they must meet pairwise in three points. The region inside the three line segments between these three intersection points forms a triangle.
Induction Hypothesis: Assume that for some $n \geq 3$, any placement of n lines in the plane in general position forms at least one triangle. Consider any placement of $n+1$ lines in the plane, and let l be any one of the lines. By the induction hypothesis, the other n lines form at least one triangle. Let τ be such a triangle. If l does not intersect τ, then τ is still a triangle after the placement of the $n+1$ lines. If l does intersect τ, then it intersects two of τ's lines in τ (call them l_1 and l_2). The region with the boundary line segments of l, l_1 and l_2 within the region of their pairwise intersection must be a triangle.

3. 12*2*3=72

4. $P(26,3) = 26*25*24 = 15{,}600$

5. The number of bitstings is the number of ways to fill out the middle n-2 bits with 0s and 1s. That is, 2^{n-2}.