CS2022/MA2201
HW#5 SOLUTIONS

1. (*a*) $P(1)$ is 1*1!=2!-1=1 which is true.
(*b*) $P(4)$ is $1*1!+2*2!+3*3!+4*4!=5!-1$ which is equvalent to $1+4+18+96=119$ which is true.

(*c*) $P(n)$ is $\sum_{k=1}^{n} k*k!=(n+1)!-1$

(*d*) $P(n+1)$ is $\sum_{k=1}^{n+1} k*k!=(n+2)!-1$

(*e*) The basis, $P(1)$, was shown in (*a*). As an induction step assume $P(n)$, that is, assume that for $n \geq 1$, $\sum_{k=1}^{n} k*k!=(n+1)!-1$. Then $\sum_{k=1}^{n+1} k*k!=\sum_{k=1}^{n} k*k!+(n+1)(n+1)!$. By the induction hypothesis, this is equal to

$(n+1)!-1+(n+1)(n+1)!=(n+1)!+(n+1)(n+1)!-1=(n+2)(n+1)!-1=(n+2)!-1$

which is $P(n+1)$.

2. <u>Basis</u> $1^2+1=2$ so $2 \mid (1^2+1)$

<u>Induction Step</u>: Assume that for any fixed positive integer n, $2 \mid (n^2+n)$. Substituting $n+1$ for n yields $(n+1)^2+(n+1)=(n^2+n)+2(n+1)$. By the Induction Hypothesis, the first term, n^2+n, is even, and clearly $2 \mid 2(n+1)$, so $2 \mid \left((n+1)^2+(n+1)\right)$, which completes the proof.

3. 4*6=24

4. (*a*) 2^{12} (*b*) 2^9 (*c*) 2^8