CS2022/MA2201
HW#5 SOLUTIONS

1. ***a*** yes

b By the rule of product, we iterate (134 times) the Cartesian Product of the set of all possible birthdays, yielding 365^{135}=

8119638639759038396031108394800622594609068082026720196758051139 36 \
5459367830703455849048414147015004771623803344476177591662418 57 \
98906330738498230322567150611678626065696085699238124046459278 7 \
46464934802473603381483368529299733750735248279409745276321497 5 \
31823603769175771405974497128417051051922491449646182570809571 6 \
4362189359962940216064453125

c $P(365,135) = 365*364*...*231 =$

3235657174392314470609537177527855800340756933312457593584985960 54 \
9524783448945461022631438745281900784579665664260355616084765 60 \
1483859586478007747382128243132749312227512592966776756778414 30 \
3086905536817450944031273869756055919710714521086794419141889 74 \
2102892172304381818530065511065332231295729664000000000000000000 \
0000000000000000

By the way, the probability of all 135 people having different birthdays is

.3984976817 10^{-12}

2. ***a*** $|A| = 26^8 =$ 208827064576

b 26^4= 456976

c There are 8 places to put the *x*, and for each placement of an *x* there are 25^7 ways to fill the other places. Hence, the answer is 8*25^7= 48828125000

d There are $\binom{8}{4}$ ways to choose the places for the *x*s, and 25^4 ways to fill the other places. Hence, the answer is $\binom{8}{4}25^4 =$ 27343750

3. A total of 12 steps must be taken, and 8 of them to be *right*. Any such choice constitutes a path, hence there are $\binom{12}{8} = \binom{12}{4} =$ 495 paths.

4. Each hand is counted twice. that is, we count “a pair of kings and a pair of eights” and we count “a pair of eights and a pair of kings”. To get the correct answer, we must divide by 2, yielding $\frac{13\binom{4}{2}12\binom{4}{2}}{2} = 2808$.