

CS2022/MA2201
HW#5

DUE: Monday, September 29

1. (3 points) Suppose that there is a group of 32 men and n women. Each man knows exactly 5 women, and each woman knows exactly 8 men. What is the value of n ?
2. (8 points) Do **EXERCISE 4.1.28** on page 311 of our text.
3. (3 points) There is a **true-false** test with 45 questions, and you know that the answer to 15 of the questions is **true**. If you're considering answering with 15 **true**'s, in how many ways can you answer the questions?
4. (4 points) If 5 points are placed inside an equilateral triangle with side length 1, prove that at least 2 of the 5 points must be at a distance from each other of at most $\frac{1}{2}$. (Hint: partition the triangle into subtriangles.)
5. (4 points) In how many ways can n rooks be placed on an $n \times n$ chessboard such that no pair of rooks attack each other? Explain your answer. By the way, two rooks attack each other if and only if they're on the same column or the same row.
6. (5 points) How many 8 letter names have a letter which occurs more than once? We only consider lower-case letters.

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HW#5 SOLUTIONS

1. Each of the 32 men belongs to exactly 5 (man,woman) pairs, so there are 160 pairs (by the rule of product). Also by the rule of product, $160=n*8$, so $n=20$.

2. (a) $26^8 = 208827064576$

(b) $P(26,8)=26*25*24*23*22*21*20*19=62990928000$

(c) $26^7 = 8031810176$

(d) $P(25,7)=25*24*23*22*21*20*19=2422728000$

(e) $26^6 = 308915776$

(f) $26^6 = 308915776$

(g) $26^4 = 456976$

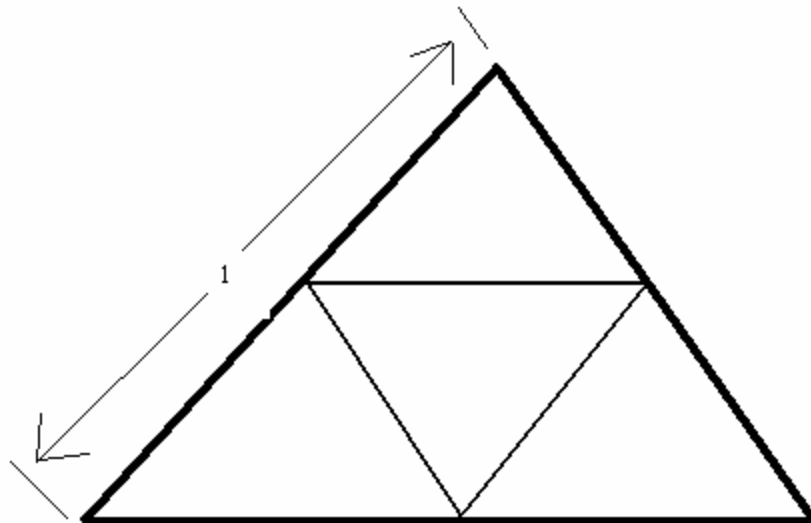
(h) $26^6 = 308915776$ start with BO, $26^6 = 308915776$ end with BO, and $26^4 = 456976$

start **and** end with BO. By the Principle of Inclusion-Exclusion,

$$26^6 + 26^6 - 26^4 = 308915776 + 308915776 - 456976 = 617374576 .$$

3. $C(45,15) = C(45,30)$

4. Consider the partition of the triangle into 4 internal regions.



The sides of the 4 internal triangles are all $\frac{1}{2}$, so no 2 points in any internal triangle are at a distance greater than $\frac{1}{2}$. By the Pigeonhole Principle, any placement of 5 points inside the large

triangle must have at least 2 points in some internal triangle, and hence the distance between them must be at most $\frac{1}{2}$.

5. In any placement, there is exactly one rook on each row. There are n squares in which to place the rook on the first row, and it eliminates its column from each of the other row. Thus, there are $n-1$ ways to place the rook on the second row, which ultimately yields $P(n, n) = n!$ ways to place the n rooks.

6. There are $26^8 = 208,827,064,576$ 8-letter names, and

$$P(26, 8) = 26 * 25 * 24 * 23 * 22 * 21 * 20 * 19 = 62,990,928,000$$

of them have distinct letters. Hence,

$$208,827,064,576 - 62,990,928,000 = 145,836,136,576$$

of them do not have distinct letters.