

CS2022/MA2201
HW#5

DUE: Monday, October 1

1. (6 points) Assume that nobody in CS2022/MA2201 is born on a leap year, and there are 135 people in the class.

a Is it possible that no two people in this class are born on the same day?

b In how many ways can we assign birthdays to these people?

c In how many ways can we assign birthdays to these people so that no two people have the same birthday?

2. (16 points) Let A be the set of all strings of length 8, where a string is a sequence of symbols from $\{a, b, \dots, z\}$ (that is, there are 26 letters of our alphabet). Thus, $abracadr \in A$ but $abracadab \notin A$ and $magic \notin A$.

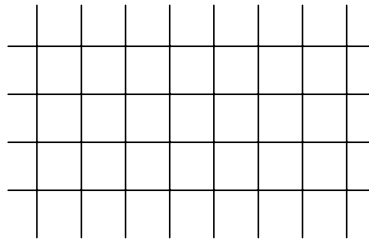
a What is $|A|$?

b How many strings in A have the prefix $abra$?

c How many strings in A have exactly one x ?

d How many strings in A have exactly four x s?

3. (4 points) Someone would like to proceed from the bottom left square below to the top right square by a sequence of steps, where each step is *up* or *right*. One path is $(right, right, up, right, up, up, right, right, right, right, up)$.



How many paths are there from the bottom left square below to the top right square?

4. (4 points) Consider the following argument:

Problem: Find the number of ways to get 2 pairs of 2 different ranks (such as 2 kings and 2 eights) in a 4-card hand from an ordinary deck of 52 cards.

Solution: There are 13 ways to get a rank (such as kings) for the first pair and $\binom{4}{2}$ ways to get a pair of that rank. Likewise, there are 12 ways to get a rank (such as

eights) for the second pair and $\binom{4}{2}$ ways to get a pair of that rank. Therefore,

there are $13\binom{4}{2}12\binom{4}{2} = 5616$ ways to get 2 pairs.

Find the error in the above solution and correct it.