CS2022/MA2201
HW#4 SOLUTIONS

1. For all $x \geq 5$, $17 \leq 2^x$, so $2^x + 17 \leq 2^x + 2^x = 2*2^x \leq 2*3^x$. So choosing C=2 and k=5 yields that 2^x+17 is $O(3^x)$.

2. Since $(\forall x \geq 1)\, x^3 \leq x^4$, then x^3 is $O(x^4)$. If x^4 were $O(x^3)$, then there'd exist C, k such that $x^4 \leq Cx^3$ for all $x \geq k$. But this would imply $x \leq C$ for all $x \geq k$, which is impossible (just choose $x = 1 + \max(k, C)$).

3. The conjecture is true. Choosing k=1 and C=2, it follows that $f(n) + g(n) \leq g(n) + g(n) = 2g(n)$ for all $n \geq 1$.

4. For each weighing, put 1/3 of the coins on the left pan and 1/3 on the right. If the coins in the left pan weigh less, then they contain the counterfeit. If the coins in the left pan weigh more, then the right pan contains the counterfeit. Otherwise the counterfeit is among the 1/3 of the coins not on the scale. In the worst case (or the best case), the algorithm uses $\lceil \log_3 n \rceil$ weighings, where n is the initial number of coins. For n=27, this algorithm uses 3 weighings.

5. (a) Modus Tollens $\Rightarrow$ "I am not sore."
Modus Tollens $\Rightarrow$ "I did not play hockey."
(b) nothing
(c) Universal Instantiation + Modus Ponens $\Rightarrow$ "Dragonflies have 6 legs."
Universal Instantiation + Modus Tollens $\Rightarrow$ "Spiders are not insects."
Existensial Generalization $\Rightarrow$ "There exists a non-6-legged creature that eats a 6-legged creature."
Existensial Generalization $\Rightarrow$ "There exists a non-insect that eats an insect."
(d) Universal Instantiation $\Rightarrow$ "If Homer is a student then he has an Internet account."
Universal Instantiation $\Rightarrow$ "If Maggie is a student then she has an Internet account."
Modus Tollens $\Rightarrow$ "Homer is not a student."
(f) Disjunctive Syllogism $\Rightarrow$ "I am hallucinating."
Modus Ponens $\Rightarrow$ "I see elephants running down the road."

6. Let x and $y$be any rational numbers. Then there exist a, b, c and c such that x=a/b and y=c/d and $b \neq 0$ and $d \neq 0$. Then $x + y = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$. Since ad+bc and bd are integers and $bd \neq 0$, then x+y must be a rational number.