CS2022/MA2201
HW#4 SOLUTIONS

1. ***a*** No, the argument is invalid in an interpretation in which p and q are false but r is true.
b No, the argument is invalid in an interpretation in which p, q and r are all true.

2. Let $P(n)$ denote the predicate $2 \mid \left(n^2 + 5n\right)$. As a basis, we note that $P(0)$ corresponds to the fact that $2|0$. Assume $P(n)$ for some fixed $n \geq 0$.

$$(n+1)^2 + 5(n+1) = n^2 + 7n + 6 = \left(n^2 + 5n\right) + 2(n+3)$$

By the Induction Hypothesis $2 \mid \left(n^2 + 5n\right)$, and clearly $2 \mid 2(n+3)$. Thus, $2 \mid \left(\left(n^2 + 5n\right) + 2(n+3)\right)$ which implies $2 \mid \left((n+1)^2 + 5(n+1)\right)$. But this is $P(n+1)$. Hence $P(n) \rightarrow P(n+1)$, and the theorem is proved.

3. As a basis, we note that any three lines in general position must meet in three points, and they form a triangle. Fix $n \geq 3$ and assume that n lines form a triangle Δ. There are two possibilities:

- If the $(n+1)^{st}$ line (call it l^*) does not intersect Δ, then the theorem holds.
- If line l^* does intersect Δ, then it intersects exactly two of the lines (call them l_1 and l_2). The points where l^* intersects l_1 and l_2, as well as the intersection of l_1 and l_2 form a triangle.

In either of these two cases, the n+1 lines form a triangle.