CS2022/MA2201
HW#4

DUE: Tuesday, April 3

1. (6 points) Prove or give a counterexample to the following.

CONJECTURE: For any functions $g : A \to B$ and $f : B \to C$, if $f \circ g$ is one-to-one, then g is one-to-one.

2. (8 points) Consider the function λ which associates with each finite bit string (possibly empty string of 0's and 1's) the number of bits in the string. For example, $\lambda(001110) = 6$, and λ of the empty string is 0. Justify your answers to parts ***b***, ***c*** and ***d***.

a What are the domain and codomain of λ?
b Is λ one-to-one?
c Is λ onto?
d Is λ a bijection?

3. (6 points) For each of the following sets, tell whether it is finite, countably infinite or uncountably infinite.

a $\{x \in \mathbb{Q} \mid 17.2 < x \leq 42\}$

b The set of all English sentences with no more than 1050 words.

c $\{x \in \Re \mid 2100 < x\}$

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HW#4 SOLUTIONS

1. The CONJECTURE is true. Assume that $f \circ g$ is one-to-one. If g were not one-to-one, then there would be $a_1, a_2 \in A$, $a_1 \neq a_2$ such that $g(a_1) = g(a_2)$.But in that case $f(g(a_1)) = f(g(a_2))$, which implies that $(f \circ g)(a_1) = (f \circ g)(a_2)$, which contradicts the assumption that $f \circ g$ is one-to-one.

2. ***a*** The domain of λ is the set of all bit strings, and the codomain is the set of all nonnegative integers, or natural numbers, $\mathbb{Z}^+ \cup \{0\} = \mathbb{N}$.

b λ is not one-to-one since $\lambda(01) = \lambda(10) = 2$ although $01 \neq 10$.

c λ is onto since for each $n \in \mathbb{Z}^+ \cup \{0\}$, $\lambda(1^n) = n$, where 1^n is the string of n 1's, and 1^0 is the empty bit string.

d Because λ is not one-to-one, it is not a bijection.

3. ***a*** countably infinite ***b*** finite ***c*** uncountably infinite