CS2022/MA2201
HW#3 SOLUTIONS

1. **a**) 4, since $f(x)$ is $O(x^4)$.
b) 5, since $f(x)$ is $O(x^5)$.
c) 0, since $f(x)$ is $O(x^0)$ or $O(1)$.
d) -1, since $f(x)$ is $O(x^{-1})$.

2. If $f(x)$ is $O(x)$, then there exist C and k such that $|f(x)| \le C|x|$ for all $x>k$. Choosing $k' = \max(k,1)$, we see that for all $x>k'$ it follows that $|f(x)| \le C|x| \le C|x^2|$, so $f(x)$ is $O(x^2)$.

3. The conjecture is false. Consider $f(x)=x$ and $g(x)=x^2$. Choosing constants C=1 and k=1, it follows that for all $n \ge k$, $f(n) = n \le n^2 = C * g(n)$. If $g(n)$ were $O(f(n))$, then there'd exist constants C, k such that for all $n \ge k$, $g(n) = n^2 \le Cn = Cf(n)$, which would imply that there exists a constant C such that $n \le C$ for all $n \ge k$, which is clearly a contradiction.

4. We saw in class that $\sum_{k=0}^{n} k = \sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$. Choosing C=1 and k=1, it follows that for all $n \ge k$, $\sum_{k=0}^{n} k = \frac{n^2}{2} + \frac{n}{2} \le \frac{n^2}{2} + n$