CS2022/MA2201
HW#3 SOLUTIONS

1. We can conclude that $A = \varnothing \vee B = \varnothing$

2. (*a*) true
(*b*) true
(*c*) true
(*d*) false Let *S*={1}, *T*={2},
$$P(S) \cup P(T) = \{\varnothing,\{1\}\} \cup \{\varnothing,\{2\}\} = \{\varnothing,\{1\},\{2\}\} \neq \{\varnothing,\{1\},\{2\},\{1,2\}\} = P(\{1,2\}) = P(S \cup T)$$
That is, $\{1,2\} \notin P(S) \cup P(T)$ but $\{1,2\} \in P(S \cup T)$.
(*e*) true
(*f*) true

3. (*a*) 1 (*b*) 2 (*c*) –1 (*d*) 0 (*e*) 3 (*f*) –2 (*g*) 1 (*h*) 2

4. (*a*) false If *x*=2.5, then $\lfloor \lceil 2.5 \rceil \rfloor = \lfloor 3 \rfloor = 3 \neq 2 = \lfloor 2.5 \rfloor$
(*b*) true

5. (*a*) yes $f^{-1}(x) = (4-x)/3$
(*b*) no, since *f*(1)=*f*(-1)
(*c*) no, since *x*=-2 is not in the domain of *f*, though it is in the range, $f\left(-\frac{5}{3}\right) = -2$.
(*d*) yes $f^{-1}(x) = \sqrt[5]{x-1}$

6. (*a*) $f(n) = \sum_{i=2}^{n}\sum_{k=1}^{i} 1 = \sum_{i=2}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{1} i = \frac{n(n+1)}{2} - 1$

(*b*) $g(n) = \sum_{i=1}^{n}\left(\frac{1}{2}\right)^i = \sum_{i=0}^{n}\left(\frac{1}{2}\right)^i - \sum_{i=0}^{0}\left(\frac{1}{2}\right)^i = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} - 1 = 1 - \left(\frac{1}{2}\right)^n$