1. a yes (every number has a unique decomposition into primes)
   b no There do not exist integers x and y such that \( f(x, y) = 2^3 3^2 = 11 \).
   c no 16 = 2^4 4^0 = f(4, 0) = 2^2 2^1 = f(2, 1) = 2^2 4^2 = f(0, 2)
   b no There do not exist integers x and y such that \( f(x, y) = 2^4 3^2 = 11 \).

2. The conjecture is false. Let \( f(x) = 2x \) and \( g(x) = x + 5 \). Since \( g \circ f(x) = 2x + 5 \) and \( f \circ g(x) = 2x + 10 \), it follows that \( g \circ f(x) \) never equals \( f \circ g(x) \) (unless 5 = 10).

3. The conjecture is true. The rational number \( \frac{p_1 q_2 + p_2 q_1}{2q_1 q_2} \) satisfies

   \[
   \frac{p_1}{q_1} < \frac{p_1 q_2 + p_2 q_1}{2q_1 q_2} < \frac{p_2}{q_2} \quad \text{(in fact it’s halfway between } \frac{p_1}{q_1} \text{ and } \frac{p_2}{q_2} \text{) and then we can}
   \]

   continue the same process to produce an unbounded number of rationals between \( \frac{p_1}{q_1} \) and

   \[
   \frac{p_1 q_2 + p_2 q_1}{2q_1 q_2}.
   \]

4. a \( f(n) = \sum_{i=4}^{n} \sum_{i=1}^{n-3} 2 = \sum_{i=4}^{n} 2(i-2) = 2 \sum_{i=4}^{n} i - 4 \sum_{i=4}^{n} 1 \). Before evaluating the sum further, we note that \( f(1) = f(2) = f(3) = 0 \). For \( n \geq 4 \),

   \[
   f(n) = 2 \left( \sum_{i=1}^{n} i - 3 \sum_{i=1}^{n} i \right) - 4(n-3) = 2 \left( \frac{n(n+1)}{2} - 6 \right) - 4n + 12 = n^2 - 3n
   \]

   Defined over the entire domain,

   \[
   f(n) = \begin{cases} 0, & \text{if } 1 \leq n \leq 3 \\ n^2 - 3n, & \text{if } 4 \leq n \end{cases}
   \]

   b \( \sum_{i=2}^{n} \left( \frac{3}{7} \right)^i = \sum_{i=0}^{n} \left( \frac{3}{7} \right)^i - \sum_{i=0}^{1} \left( \frac{3}{7} \right)^i = \frac{1 - \left( \frac{3}{7} \right)^{n+1}}{1 - \frac{3}{7}} - \frac{10}{7} = \frac{9}{28} - \frac{7}{4} \left( \frac{3}{7} \right)^{n+1} \)