CS2022/MA2201
HW#3

DUE: Monday, September 13

1. (3 points) Do **Exercise 1.4.20** from our text.

2. (12 points) Let the universe of discourse be $U = P(\mathrm{Z})$, where Z={...,-2,-1,0,1,2,...}. Tell whether each of the following statements is true or false. If the statement is false, give an example of sets *S* and *T* which provide a counterexample and show that your sets are a counterexample.

(*a*) $(\forall S)(\forall T)(S \subseteq T \to S \cap T = S)$

(*b*) $(\forall S)(\forall T)(S \subseteq T \to S - T = \varnothing)$

(*c*) $(\forall S)(\forall T)(P(S) \cup P(T) \subseteq P(S \cup T))$

(*d*) $(\forall S)(\forall T)(P(S) \cup P(T) = P(S \cup T))$

(*e*) $(\forall S)(\forall T)(P(S) \cap P(T) = P(S \cap T))$

(*f*) $(\forall S)(\forall T)(|S \times T| = |T \times S|)$

3. (8 points) Do **Exercise 1.6.6** from our text.

4. (4 points) Let the universe of discourse be $U = \mathrm{R}$, the set of real numbers. Are the following statements true or false? If the statement is false, give an example of $x \in \mathrm{R}$ which provides a counterexample.

(*a*) $(\forall x)\lfloor \lceil x \rceil \rfloor = \lfloor x \rfloor$

(*b*) $(\forall x)\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$

5. (8 points) Do **Exercise 1.8.8** from our text. If *f*(*x*) is a bijection, then describe $f^{-1}(x)$.

6. (6 points) (*a*) Give a closed form (a form which doesn't have summation symbols) for the function $f(n) = \sum_{i=2}^{n} \sum_{k=1}^{i} 1$ where $f : \mathrm{Z}^+ \to \mathrm{Z}$. Be careful to give a correct answer for the entire domain of *f*.

(*b*) Give a closed form (a form which doesn't have summation symbols) for the function $g(n) = \sum_{i=1}^{n} \left(\frac{1}{2}\right)^i$ where $g : \mathrm{Z}^+ \to \mathrm{Z}$.