CS2022/MA2201
HW#3

DUE: Thursday, March 29

1. (6 points) Tell whether each of the following is true or false.

a $4 \in \left\{ x \in \Re \mid \exists y \in \mathbb{Z}\left(x = y^2 \right) \right\}$

b $4 \subseteq \left\{ x \in \Re \mid \exists y \in \mathbb{Z}\left(x = y^2 \right) \right\}$

c $4 \in \{\{4\}\}$

d $4 \subseteq \{4\}$

e $\{\varnothing\} \in \{\varnothing\}$

f $\{\varnothing\} \subseteq \{\varnothing\}$

2. (2 points) ***a*** What is $\left|\{2, \{\varnothing\}, \varnothing\}\right|$?

b What is $\left|\{2, \{2, \{2\}\}\}\right|$?

3. (3 points) ***a*** Find a set *A*, if there is one, such that the power set of *A* is $\{\varnothing\}$.

b Find a set *A*, if there is one, such that the power set of *A* is $\varnothing$.

c Find a set *A*, if there is one, such that the power set of *A* is $\{\varnothing, \{1\}, \{1, 2\}\}$.

4. (4 points) Prove or give a counterexample to the following.

> **CONJECTURE**: For any sets *A* and *B*, if $A \times B = \varnothing$, then $A = \varnothing$ and $B = \varnothing$.

5. (4 points) (Exercise 34 of Section 2.1 of the text.) Determine whether each of the following is true or false.

a $\exists x \in \Re \left(x^3 = -1 \right)$

b $\exists x \in \mathbb{Z} \left(x + 1 > x \right)$

c $\forall x \in \mathbb{Z} \left(x - 1 \in \mathbb{Z} \right)$

d $\forall x \in \mathbb{Z} \left(x^2 \in \mathbb{Z} \right)$

6. (4 points) Tell whether each of the following is true or false for all sets *A* and *B*, and justify your answer.

a $(A \cap B) \subseteq A$

b $(A \cup B) \subseteq A$

c $(A \cap B) \subseteq (A \cup B)$

d $((A \cap B) = (A \cup B)) \to (A = B)$

7. (5 points) Let *A* be the set of all nonempty bit strings. That is, $A = \{0, 1, 00, 01, 10, 11, 000, 001, ...\}$. Let function *f* associate with any bit string *x* the length of the longest string of consecutive 1's in *x*. For example, $f(00101) = 1$, $f(000) = 0$ and $f(011100011011101) = 3$. Is $\{n \mid \exists x \in A(f(x) = n)\}$ finite or infinite? Justify your answer.

CS2022/MA2201
HW#3 SOLUTIONS

1. ***a*** true ***b*** false ***c*** false ***d*** false ***e*** false ***f*** true

2. ***a*** 3 ***b*** 2

3. ***a*** $A = \varnothing$ ***b*** There is no such set *A*. ***c*** There is no such set *A*.

4. The CONJECTURE is false. As a counterexample, if $A = \{1\}$ and $B = \varnothing$ then $A \times B = \varnothing$ although $A \neq \varnothing$. In fact, the most we can conclude is that $A = \varnothing$ or $B = \varnothing$

5. ***a***, ***b***, ***c*** and ***d*** are all true.

6. ***a*** true For any $x \in U$, if $x \in A \cap B$ then $x \in A$
b false If $A = \varnothing$ and $B = \{1\}$, then $A \cup B = \{1\}$, which is not a subset of $\varnothing$.
c true $A \cap B = \{x \mid x \in A \wedge x \in B\} \subseteq \{x \mid x \in A \vee x \in B\} = A \cap B$

7. $\{n \mid \exists x \in A(f(x) = n)\}$ is infinite. In fact, it is $\mathbb{Z}^+ \cup \{0\}$. Clearly $\{n \mid \exists x \in A(f(x) = n)\} \subseteq \mathbb{Z}^+ \cup \{0\}$, and $f(1^n) = n$ shows that every $n \in \mathbb{Z}^+$ belongs to $\{n \mid \exists x \in A(f(x) = n)\}$. Likewise, because $f(0) = 0$ it follows that $0 \in \{n \mid \exists x \in A(f(x) = n)\}$.