CS2022/MA2201
HW#2 SOLUTIONS

1. (*a*) true ($x=\sqrt{2}$)
(*b*) false ($\sqrt{-1}\notin\Re$)
(*c*) true
(*d*) false (in case $x<0$)
(*e*) true ($x=0$)
(*f*) false (+ is commutative)
(*g*) true ($y=1/x$)
(*h*) false
(*i*) true ($x=y=1/2$)
(*j*) false
(*k*) false ($x=0\Rightarrow y=2\Rightarrow$ second equation incorrect)
(*l*) true ($z=(x+y)/2$)

2. (*a*) $(\forall y)(\forall x)\neg P(x,y)$
(*b*) $(\exists x)(\forall y)\neg P(x,y)$
(*c*) $(\forall y)(\neg Q(y)\vee(\exists x)R(x,y))$
(*d*) $(\forall y)((\forall x)\neg R(x,y)\wedge(\exists x)\neg S(x,y))$
(*e*) $(\forall y)((\exists x)(\forall z)\neg T(x,y,z)\wedge(\forall x)(\exists z)\neg U(x,y,z))$

3.(*a*) $U=\{1,2\}$, $P=\{(1),(2)\}$, $Q=\varnothing$
(*b*) $U=\{1,2\}$, $P=Q=\varnothing$

4. (*a*) no For all sets S, $\varnothing\in P(S)$, hence $\varnothing$ can not be the power set of any set, that is, $\neg(\exists S)\varnothing=P(S)$.
(*b*) yes $P(\{a\})=\{\varnothing,\{a\}\}$
(*c*) no If $|S|=n$, then $|P(S)|=2^n$, if a set is a power set of a set, then its cardinality is a power of 2. Since $|\{\varnothing,\{a\},\{\varnothing,a\}\}|=3$, it can not be the power set of a set.
(*d*) yes $P(\{a,b\})=\{\varnothing,\{a\},\{b\},\{a,b\}\}$

5. (*a*) No, since $2\in R\wedge 2\notin S$
(*b*) No, since $4\in R\wedge 4\notin T$
(*c*) Yes, since $(\forall x)6|x\rightarrow 2|x$
(*d*) Yes, since $(\forall x)6|x\rightarrow 3|x$
(*e*) $R\cap S\cap T=T$

6. The statement is true. $A \subseteq B$ if and only if $(\forall x)\, x \in A \rightarrow x \in B$ if and only if $\neg(\exists x)\, x \in A \wedge x \notin B$ if and only if $A - B = \varnothing$.