CS2022/MA2201
HW#2 SOLUTIONS

1. The first statement is false and the second statement is true in the interpretation with universe of discourse $U=\{a,b\}$ with $P(a)$, $\neg P(b)$, $\neg Q(a)$ and $Q(b)$. That is, $P=\{(a)\}$ and $Q=\{(b)\}$.

2. The CONJECTURE is true. One method of proof is to use a membership table.

A	B	C	$A\oplus B$	$(A\oplus B)\oplus C$	$B\oplus C$	$A\oplus(B\oplus C)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	0	0	0
1	0	0	1	1	0	1
1	0	1	1	0	1	0
1	1	0	0	0	1	0
1	1	1	0	1	0	1

In general, for any *x* and any sets $A_1,...,A_n$, $x\in A_1\oplus A_2\oplus...\oplus A_n$ (it doesn't matter where you put the parentheses) if and only if *x* belongs to an odd number of A_i. A string of $\oplus$ operators checks parity.

3. ***a*** The CONJECTURE is false. A counterexample is the sets $A=\{1,2,3\}$, $B=\{1,2,4\}$ and $C=\{2,3,4\}$. $|A\cap B|=|A\cap C|=|B\cap C|=2$ although $|(A\cap B)\cap C|=|\{2\}|=1$.

b The CONJECTURE is false. A counterexample is the sets $A=\{1\}$, $B=\{2\}$ and $C=\{1,2\}$.

c The CONJECTURE is false. A counterexample is the sets $A=\{1\}$, $B=\{2\}$ and $C=\varnothing$.

4. ***a*** $\{x\,|\,x\in\mathbb{Z}\wedge x^2<10\}=\{-3,-2,-1,0,1,2,3\}$ which has cardinality 7.

b 8

c 675

d 0