CS2022/MA2201
HW#2

DUE: Friday, March 23

1. (12 points) For each of the following arguments, tell whether the argument is correct or not, and explain why. Justify every step of your proof in case the argument is correct. Note that an argument can be correct even if some of its premises are not. That is, the argument in ***a*** can be correct even if there are Brazilians who don't play soccer.
a Every Brazilian excels in soccer. Nelio does not play soccer well. Therefore Nelio is not Brazilian.
b Everybody likes easy classes. MA 2201 is hard. Therefore Deb does not like MA 2201.
c Every happy person smiles a lot. Therefore somebody smiles a lot.

2. (6 points) Prove the following, which is Exercise 14 of Section 1.7 of our text. As arithmetic rules you may note that for all *x*, *y* and *u*, if *x*=*y* then $x+u=y+u$, $x-u=y-u$ and *x***u*=*y***u*, and if $u \neq 0$ then $x/u = y/u$.

THEOREM: For any $a,b,c \in \Re$, $a \neq 0$, there is exactly one unique solution to the equation $ax+b=c$. That is, there is exactly one value of *x* which satisfies the equality.

3. (5 points) For any $p,q \in \mathbb{Z}^+$ we write $p \mid q$ if there exists $r \in \mathbb{Z}^+$ such that $p * r = q$. A number $n \in \mathbb{Z}^+$ is a *Maia number* if $\sum_{\substack{1 \le m < n \\ m \mid n}} m = n$. Do there exist at least two distinct Maia numbers? Justify your answer.

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HW#2 SOLUTIONS

1. ***a*** The argument is correct. We let unary predicates *B*(*x*) denote that *x* is Brazilian and *E*(*x*) denote that *x* excels at soccer. *Nelio* is a constant. The argument can be rephrased as

$$\forall x\, B(x) \to E(x)$$
$$\neg E(Nelio)$$
$$\overline{\qquad\qquad\qquad}$$
$$\neg B(Nelio)$$

We use universal instantiation to go from $\forall x\, B(x) \to E(x)$ to $B(Nelio) \to E(Nelio)$. We use modus tolens to go from $\neg E(Nelio)$ and $B(Nelio) \to E(Nelio)$ to our conclusion, $\neg B(Nelio)$. So the total proof is

$$\forall x\, B(x) \to E(x)$$
$$B(Nelio) \to E(Nelio)$$
$$\neg E(Nelio)$$
$$\overline{\qquad\qquad\qquad}$$
$$\neg B(Nelio)$$

b The argument is not correct. We let unary predicate *Easy*(*z*) denote that class *z* is easy, and binary predicate *Likes*(*x*,*z*) denote that person *x* likes class *z*. Our domain of discourse has one person, *Deb*, and one class, *MA 2201*. The predicate *Easy* is empty; there are no easy classes, and the predicate *Likes* is the set $\{(Deb, MA2201)\}$; that is, *Deb* likes *MA 2201*. The premises of the argument are each true in this interpretation, but the conclusion is false.
c The argument is not correct because it is false in the interpretation in which nobody is happy and nobody smiles.

2. For a constructive solution, we note that $ax + b = c$ implies that $ax = c - b$, which implies that $x = (c - b)/a$.

For a proof by contradiction, we assume that there exist distinct values *y* and *z*, $y \neq z$, such that $ay + b = c = az + b$. Inferring that $ay + b = az + b$, we note that this implies that $ay = az$, which implies that *y*=z, which contradicts that $y \neq z$. Hence, distinct solutions can not exist.

3. Because $\{m | 1 \leq m < 6 \wedge m \mid 6\} = \{1, 2, 3\}$ and 6=1+2+3, it follows that 6 is a Maia number. Because $\{m | 1 \leq m < 28 \wedge m \mid 28\} = \{1, 2, 4, 7, 14\}$ and 28=1+2+4+7+14, it follows that 28 is a Maia number. We have proved constructively that there exist at least two Maia numbers. These numbers are properly called *perfect numbers*.