CS2022/MA2201
HW#2

DUE: Tuesday, September 9

1. (6 points) Do **Exercise 1.3.34** from the text.

2. (3 points) Do **Exercise 1.4.22** from the text, where the the universe of discourse is the positive integers.

3. (9 points) Do **Exercise 1.4.26** from the text.

4. (6 points) (*a*) Give an interpretation in which $(\exists x)\big(P(x) \vee (\forall y) Q(x, y)\big)$ is true.
(*b*) Give an interpretation in which $(\exists x)\big(P(x) \vee (\forall y) Q(x, y)\big)$ is false.

5. (5 points) Do **Exercise 1.5.2** from the text.

6. (12 points) Do **Exercise 1.5.12** from the text.

CS2022/MA2201
HW#2 SOLUTIONS

1. (a) x=1 is a counterexample.
(b) $x=\sqrt{2}$ and $x=-\sqrt{2}$ are the two counterexamples.
(c) x=0 is a counterexample.

2. $(\exists x)\neg(\exists y)(\exists z)(\exists u)\big((x>0)\wedge x=y^2+z^2+u^2\big)$

3. (a) false (b) true (c) false (d) false (e) true (f) true (g) true (h) false (i) false

4. Let the universe of discourse be $U=\{a,b\}$.
(a) For any interpretation in which predicate P holds for a, the statement is true.
(b) In the interpretation in which predicate P does not hold for a or b, and Q does not hold for any pair of objects, the statement is false.

5. (a) simplification
(b) disjunctive syllogism
(c) modus ponens
(d) addition
(e) hypothetical syllogism

6. (a) true The hypotheses are $(\forall x)\,Enrolled(x)\rightarrow LiveinDorm(x)$ and $\neg LiveinDorm(Mia)$, and the conclusion is $\neg Enrolled(Mia)$. The argument is

$(\forall x)\,Enrolled(x)\rightarrow LiveinDorm(x)$	hypothesis
$Enrolled(Mia)\rightarrow LiveinDorm(Mia)$	Universal Instantiation
$\neg LiveinDorm(Mia)$	hypothesis
$\neg Enrolled(Mia)$	modus tolens

(b) false The hypotheses are $(\forall x)\,convertible(x)\rightarrow fun(x)$ and $\neg convertible(Isaac'sCar)$, and the conclusion is $\neg fun(Isaac'sCar)$. Let the universe of discourse be $U=\{Isaac'sCar, Ben'sCar\}$, and $fun=\{(Isaac'sCar),(Ben'sCar)\}$ and $convertible=\{(Ben'sCar)\}$. In this interpretation the hypotheses are true but the conclusion is false. That is, the argument

$$(\forall v)\, convertible(x) \rightarrow fun(x)$$
$$\neg convertible(Isaac'sCar)$$

$$\therefore \neg fun(Isaac'sCar)$$

is not a valid argument.

(c) false The hypotheses are $(\forall x)\, ActionMovie(x) \rightarrow QuincyLikes(x)$ and $QuincyLikes(8MenOut)$, and the conclusion is $ActionMovie(8MenOut)$. Let the universe of discourse be $U = \{8MenOut, StarWars\}$, and $ActionMovie = \{(StarWars)\}$ and $QuincyLikes = \{(StarWars), (8MenOut)\}$. In this interpretation the hypotheses are true but the conclusion is false. That is, the argument

$$(\forall v)\, ActionMovie(x) \rightarrow QuincyLikes(x)$$
$$QuincyLikes(8MenOut)$$

$$\therefore ActionMovie(8MenOut)$$

is not a valid argument.

(d) true The hypotheses are $(\forall x)\, LobsterMan(x) \rightarrow Set12Traps(x)$ and $Lobsterman(Hamilton)$, and the conclusion is $Set12Traps(Hamilton)$. The argument is

$(\forall x)\, LobsterMan(x) \rightarrow Set12Traps(x)$	hypothesis
$LobsterMan(Hamilton) \rightarrow Set12Traps(Hamilton)$	Universal Instantiation
$Lobsterman(Hamilton)$	hypothesis
$Set12Traps(Hamilton)$	modus ponens