CS2022/MA2201
HW#2

DUE: Tuesday, September 9

1. (6 points) Do Exercise 1.3.34 from the text.

2. (3 points) Do Exercise 1.4.22 from the text, where the universe of discourse is the positive integers.

3. (9 points) Do Exercise 1.4.26 from the text.

4. (6 points) (a) Give an interpretation in which \((\exists x)(P(x) \lor (\forall y)Q(x, y))\) is true.
   (b) Give an interpretation in which \((\exists x)(P(x) \lor (\forall y)Q(x, y))\) is false.

5. (5 points) Do Exercise 1.5.2 from the text.

6. (12 points) Do Exercise 1.5.12 from the text.
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HW#2 SOLUTIONS

1. (a) \( x=1 \) is a counterexample.
   (b) \( x = \sqrt{2} \) and \( x = -\sqrt{2} \) are the two counterexamples.
   (c) \( x=0 \) is a counterexample.

2. \( \exists x \neg(\exists y)(\exists z)(\exists u)(x > 0) \land x = y^2 + z^2 + u^2 \) 

3. (a) false (b) true (c) false (d) false (e) true (f) true (g) true (h) false (i) false

4. Let the universe of discourse be \( U = \{a, b\} \).
   (a) For any interpretation in which predicate \( P \) holds for \( a \), the statement is true.
   (b) In the interpretation in which predicate \( P \) does not hold for \( a \) or \( b \), and \( Q \) does not hold for any pair of objects, the statement is false.

5. (a) simplification
   (b) disjunctive syllogism
   (c) modus ponens
   (d) addition
   (e) hypothetical syllogism

6. (a) true The hypotheses are \( \forall x \) \( \text{Enrolled}(x) \rightarrow \text{LiveinDorm}(x) \) and \( \neg \text{LiveinDorm}(\text{Mia}) \), and the conclusion is \( \neg \text{Enrolled}(\text{Mia}) \). The argument is
   \[
   \begin{align*}
   (\forall x) \text{Enrolled}(x) & \rightarrow \text{LiveinDorm}(x) & \text{hypothesis} \\
   \text{Enrolled}(\text{Mia}) & \rightarrow \text{LiveinDorm}(\text{Mia}) & \text{Universal Instantiation} \\
   \neg \text{LiveinDorm}(\text{Mia}) & & \text{hypothesis} \\
   \neg \text{Enrolled}(\text{Mia}) & & \text{modus tolens}
   \end{align*}
   \]
   (b) false The hypotheses are \( \forall x \) \( \text{convertible}(x) \rightarrow \text{fun}(x) \) and \( \neg \text{convertible}(\text{Isaac's Car}) \), and the conclusion is \( \neg \text{fun}(\text{Isaac's Car}) \). Let the universe of discourse be \( U = \{\text{Isaac's Car, Ben's Car}\} \), and \( \text{fun} = \{(\text{Isaac's Car}), (\text{Ben's Car})\} \) and \( \text{convertible} = \{(\text{Ben's Car})\} \). In this interpretation the hypotheses are true but the conclusion is false. That is, the argument
\[(\forall v)\text{convertible}(x) \rightarrow \text{fun}(x)\]
\[\neg\text{convertible}(\text{Isaac's Car})\]

\[\therefore \neg\text{fun}(\text{Isaac's Car})\]

is not a valid argument.

c) false The hypotheses are \((\forall x)\text{ActionMovie}(x) \rightarrow \text{QuincyLikes}(x)\) and \(\text{QuincyLikes}(8\text{MenOut})\), and the conclusion is \(\text{ActionMovie}(8\text{MenOut})\). Let the universe of discourse be \(U = \{8\text{MenOut}, \text{StarWars}\}\), and \(\text{ActionMovie} = \{(\text{StarWars})\}\) and \(\text{QuincyLikes} = \{(\text{StarWars}), (8\text{MenOut})\}\). In this interpretation the hypotheses are true but the conclusion is false. That is, the argument
\[(\forall v)\text{ActionMovie}(x) \rightarrow \text{QuincyLikes}(x)\]
\[
\text{QuincyLikes}(8\text{MenOut})
\]

\[\therefore \text{ActionMovie}(8\text{MenOut})\]

is not a valid argument.

d) true The hypotheses are \((\forall x)\text{LobsterMan}(x) \rightarrow \text{Set12Traps}(x)\) and \(\text{Lobsterman}(\text{Hamilton})\), and the conclusion is \(\text{Set12Traps}(\text{Hamilton})\). The argument is
\[(\forall x)\text{LobsterMan}(x) \rightarrow \text{Set12Traps}(x)\] hypothesis
\[
\text{LobsterMan}(\text{Hamilton}) \rightarrow \text{Set12Traps}(\text{Hamilton})\] Universal Instantiation
\[
\text{Lobsterman}(\text{Hamilton})\] hypothesis
\[
\text{Set12Traps}(\text{Hamilton})\] modus ponens