CS2022/MA2201
HW#2

DUE: Tuesday, September 11

1. (4 points) Do **Exercise 1.3.40** from our text.

2. (6 points) We define an operation $\oplus$ on sets by $A \oplus B = \{x | x \in A - B\} \cup \{x | x \in B - A\} = (A - B) \cup (B - A)$ Prove or give a counterexample to the following claim that $\oplus$ is associative.
CONJECTURE: For all sets *A*, *B* and *C*, $(A \oplus B) \oplus C = A \oplus (B \oplus C)$.
(Hint: To disprove a conjecture with universally quantified variables, you only need provide instances of the variables for which the conjecture fails (this is called a counterexample). If the conjecture is true, you must prove that it's true for all possible instances of the variables, not just for the instances of your choice. That is, you can't prove the above conjecture just by showing that it holds for $A = \{1, 2\}, B = \varnothing$ and $C = \{7, 9, 255\}$.)

3. (9 points) Prove or give a counterexample to each of the following:
   a **CONJECTURE 1**: For all sets *A*, *B* and *C*, if $|A \cap B| = |A \cap C| = |B \cap C|$, then
   $$|A \cap B| = |(A \cap B) \cap C|.$$
   b **CONJECTURE 2**: For all sets *A*, *B* and *C*, if $A \cup C = B \cup C$, then $A = B$.
   c **CONJECTURE 3**: For all sets *A*, *B* and *C*, if $A \cap C = B \cap C$, then $A = B$.

4. (4 points) What is the cardinality of each of the following sets?
   a $\{x | x \in \mathbb{Z} \wedge x^2 < 10\}$
   b $P(\{Ben, Isaac, Maia\})$, where $P(A)$ returns the power set of *A*
   c $A \times B$, where *A* is the set of 135 students in this class and *B* is the set of 5 people consisting of the TAs, SLAs and professor.
   d $A \times \varnothing$, where *A* is the set of 135 students in this class.