CS2022/MA2201
HW#1 SOLUTIONS

1. **a**) $r \wedge \neg q$
b) $p \wedge q \wedge r$
c) $r \rightarrow p$
d) $p \wedge \neg q \wedge r$
e) $(p \wedge q) \rightarrow r$
f) $r \leftrightarrow (q \vee p)$ although $r \leftrightarrow (q \oplus p)$ would be acceptable

2. **a**)

p	q	r	$(p \vee q) \wedge (p \vee r)$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

b)

p	q	$(q \vee p) \leftrightarrow (q \oplus p)$
F	F	T
F	T	T
T	F	T
T	T	F

3. **a**) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ is a tautology because

p	q	r	$\big((p \to q) \wedge (q \to r)\big) \to (p \to r)$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

b) $\big((p \wedge q) \wedge (q \wedge r)\big) \to (p \wedge r)$ is a tautology because

p	q	r	$\big((p \wedge q) \wedge (q \wedge r)\big) \to (p \wedge r)$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

c) $\big((p \oplus q) \wedge (q \oplus r)\big) \to (p \oplus r)$ is not a tautology as seen by the case where p and r are T and q is F, or the case where p and r are F and q is T.

4.

p	q	$p \to q$	$\neg q \to \neg p$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

5. **a**) $(\forall x)F(x,Fred)$

b) $(\forall x)F(Evelyn,x)$

c) $(\forall x)(\exists y)F(x,y)$

d) $\neg(\exists x)(\forall y)F(x,y)$

e) $(\forall x)(\exists y)F(y,x)$

f) $\neg(\exists x)\big(F(x,Fred) \wedge F(x,Jerry)\big)$

g) $(\exists x)(\exists y)\Big(F(Nancy,x) \wedge F(Nancy,y) \wedge (x \neq y) \wedge (\forall z)\big(F(Nancy,z) \to ((z = x) \vee z = y)\big)\Big)$

h) $(\exists x)\Big(\forall y)\Big(F\big(y,x\big)\wedge(\forall z)(\forall u)\big(F(u,z)\rightarrow(z=x)\big)\Big)$

i) $\neg(\exists x)F(x,x)$

j) $(\exists x)\Big(\exists y)\Big(\big(x\neq y\big)\wedge F\big(x,y\big)\wedge(\forall z)\Big(F(x,z)\rightarrow\big(z=y\big)\Big)\Big)$