CS2022/MA2201
HW#1

DUE: Monday, March 19

1. (2 points) ***a***What is the converse of
"If Isaac sits in the back row, then he is cheating."?
bWhat is the contrapositive of
"If Isaac sits in the back row, then he is cheating."?

2. (3 points) Are $q \wedge \neg p$ and $\neg(p \vee \neg q)$ logically equivalent? Justify your answer.

3. (6 points) Suppose that $P(x)$ is the statement "$x+1=2x$". What is the truth value of the following statements when the universe of discourse is $\mathbb{Z}$?
a $\exists x P(x)$
b $\forall x P(x)$
c $\exists x (x=1) \rightarrow P(x)$

4. (6 points) Consider the proposition "If $x>0$ then $x^2>0$".
a Is the proposition true?
b What is the contrapositive of the proposition and is it true?
c What is the converse of the proposition and is it true?

5. (8 points) Suppose the universe of discourse is $\Re^+$, the positive real numbers. Tell whether each of the following statements is true or false, and tell why.
a $\exists x\, x>1$
b $\forall x\, x>1$
c $\forall x \exists y\, x>y$
d $\exists x \forall y\, x>y$

6. (6 points) Consider the compound proposition from Exercise **60 a** on pg. 30 of our text.

$$(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$$

a Is the proposition a tautology? Justify your answer.
b Is the proposition a contingency? Justify your answer.
c Is the proposition a contradiction? Justify your answer.

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HW#1 SOLUTIONS

1. ***a*** If Isaac is cheating, then he sits in the back row.
b If Isaac isn't cheating, then he doesn't sit in the back row.

2. We can use a truth table to establish that they are logically equivalent..

p	q	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$	$\neg p$	$q \wedge \neg p$
F	F	T	T	F	T	F
F	T	F	F	T	T	T
T	F	T	T	F	F	F
T	T	F	T	F	F	F

3.***a*** Binding x to 1 yields that $\exists x P(x)$ is true.

b Since $3+1 \neq 2*3$, $\forall x P(x)$ is false.

c Binding x to 42 yields that $\exists x (x=1) \rightarrow P(x)$ is true.

4.***a*** The proposition is true.
b The contrapositive of the proposition is "If $x^2 \leq 0$ then $x \leq 0$ ", and it is true. The statement $x^2 \leq 0$ is true only if x=0, in which case $x \leq 0$.
c The converse of the proposition is "If $x^2 > 0$ then $x > 0$ ", and it is false. Letting x=-3 we note that $(-3)^2 > 0$ though $\neg(-3 > 0)$.

5. ***a*** The statement is true for $x = \sqrt{2}$, and for many other choices for x!

b The statement is false, because of x=0.00396, and because of $x = \frac{1}{\sqrt{2}}$.

c The statement is true because for any x we choose $y=x/2$.
d The statement is false because it asserts that there is a real number which is larger than every other real number. In particular, for any x the statement $\forall y \; x > y$ must hold. Even for $y=x+1$.

6. ***a*** The proposition is not a tautology. It is false if p=F and q=F and r=T.
b Since the proposition is neither a tautology nor a contradiction, it is a contingency.
c The proposition is not a contradiction. It is true if p=T and q=F.