

CS2022/MA2201
HW#1

DUE: Thursday, September 4

1. (6 points) Do **Exercise 1.1.10** from our text.
2. (3 points) Do **Exercise 1.1.22** from our text.
3. (3 points) (a) Is an implication logically equivalent to its converse?
(b) Is an implication logically equivalent to its contrapositive?
(c) Is an implication logically equivalent to its inverse?
4. (5 points) Do **Exercise 1.3.10** from our text.
5. (9 points) Suppose the universe of discourse is all living people, and the predicates are:
 $M(x)$: x is a male
 $F(x)$: x is a female
 $A(x,y)$: x is an ancestor of y .
 $H(x,y)$: x is the mother of y .

Write each of the following statements using the above predicates and any needed quantifiers:

- (a) *Ben* is *Maia*'s mother.
- (b) All mothers are women.
- (c) If you are a female ancestor, then you are a mother.
- (d) There is a person who is male and female.
- (e) There is a female.
- (f) There is a male who is not a mother.

Write each of the following in good English.

- (g) $H(\textit{Isaac}, \textit{Maia})$
- (h) $\neg(\exists z)A(\textit{Isaac}, z)$
- (i) $(\forall x)((\exists y)H(y, x) \rightarrow M(x))$

6. (4 points) For each of the following, tell whether or not the statement is a tautology, contingency or contradiction.

- (a) $(\forall x)(P(x) \vee \neg P(x))$
- (b) $(\forall x)(P(x) \wedge \neg P(x))$
- (c) $(\forall x)(P(x) \rightarrow \neg P(x))$

(d) $(\forall x)(P(x) \rightarrow P(x))$

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HW#1 SOLUTIONS

1. (a) $r \wedge \neg q$
(b) $p \wedge q \wedge r$
(c) $r \rightarrow p$
(d) $p \wedge \neg q \wedge r$
(e) $(p \wedge q) \rightarrow r$
(f) $r \leftrightarrow (q \vee p)$

2. (a) **Converse:** If I stay at home tonight, then it will snow.

Contrapositive: If I go out tonight, then it will not snow.

Inverse: If it doesn't snow tonight, then I will go out.

(b) **Converse:** If I go to the beach, then it is a sunny summer day.

Contrapositive: If I do not go to the beach, then it is not a sunny summer day.

Inverse: If it is not a sunny summer day, then I do not go to the beach.

(c) **Converse:** If I sleep until noon, then I stayed up late.

Contrapositive: If I do not sleep until noon, then I did not stay up late.

Inverse: If don't stay up late, then I do not sleep until noon.

3. As stated on page 8 of our text, the contrapositive of an implication is logically equivalent to the original implication, but the converse and the inverse are not.

4. (a) $(\exists x)(C(x) \wedge D(x) \wedge F(x))$
(b) $(\forall x)(C(x) \vee D(x) \vee F(x))$
(c) $(\exists x)(C(x) \wedge F(x) \wedge \neg D(x))$
(d) $\neg(\exists x)(C(x) \wedge D(x) \wedge F(x))$
(e) $(\exists x)C(x) \wedge (\exists x)D(x) \wedge (\exists x)F(x)$

5. (a) $H(\text{Ben}, \text{Maia})$

(b) $(\forall x)((\exists y)H(x, y) \rightarrow F(x))$

(c) $(\forall x)(F(x) \wedge (\exists y)A(x, y)) \rightarrow (\exists z)H(x, z)$, or we could have written

$(\forall x)(F(x) \wedge (\exists y)A(x, y)) \rightarrow (\exists y)H(x, y)$ since the scopes of the two
existential quantifiers do not overlap.

(d) $(\exists x)(M(x) \wedge F(x))$

(e) $(\exists x) F(x)$

(f) $(\exists x)(M(x) \wedge \neg(\exists y)H(x, y))$

(g) Isaac is Maia's mother.

(h) Isaac is not anybody's ancestor.

(i) If you have a mother, then you are male or you have an ancestor.

6. (a) tautology (b) contradiction (c) contingency (d) tautology