CS2022/MA2201

Name________

Final Exam

Date: Thursday, October 16, 1997

All documentation permitted

1. (20 points) Given a sample space W of all permutations of $\{1,2,3,4\}$, define the random variable $X:\Omega\to\{1,2,3,4\}$ to denote the location of 1 in the permutation.. For example, $X((2,4,1,3))=3$.

A) What is the expected value of X if the probability of (2,1,4,3) is 1 and the probability of all other permutations is 0?

B) What is the expected value of X if the probability of each permutation is 1/24? Justify your solution.

2. (20 points) Prove by induction that $3^n \leq n!$ for all n=7. (Hint: 7!=5040 and 3^7=2187)

3. (20 points) Prove or give a counterexample to the following:
CONJECTURE: For set S and binary relation R from S to S, if R is reflexive and symmetric, then R must be transitive.

4. (20 points) Prove or give a counterexample to the following:
<u>CONJECTURE</u>: For any graphs G, H, if they have the same degree sequences, then they must be isomorphic.

5. (20 points) The US Senate has 100 members. If the senators consist of 85 men and 15 women, in how many ways can we pick a committee of 10 senators consisting of 5 men and 5 women?

CS2022/MA2201
Solutions to Final Exam

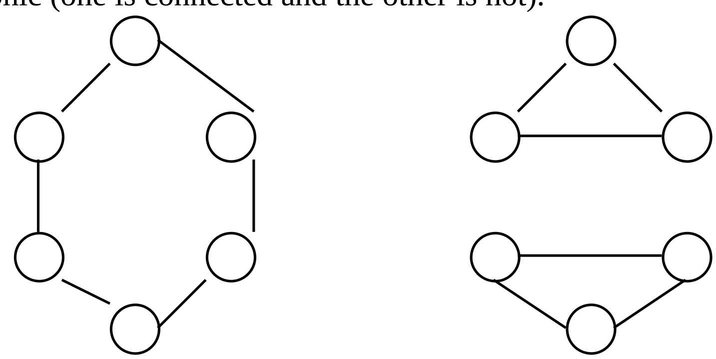
1. A) 2

B) $E[X]=\sum_{i=1}^{4} p(X=i)*i=\sum_{i=1}^{4}\frac{6}{24}*i=\frac{1}{4}\sum_{i=1}^{4} i=\frac{1}{4}\frac{4*5}{2}=\frac{5}{2}$

2) Basis: $3^7 = 2187 \le 5040 = 7!$
Induction hypothesis: Assume that for n=7, $3^n \le n!$.
Induction Step: $3^{n+1} = 3*3^n \le 3*n! \le (n+1)*n! = (n+1)!$

3) The CONJECTURE is false. Consider S={1,2,3} and
R={(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)}. R is symmetric and reflexive, but not transitive.

4) The CONJECTURE is false. The following graphs each have degree sequences (2,2,2,2,2,2) but they are not isomorphic (one is connected and the other is not).



5) There are C(85,5) ways to pick 5 men, and C(15,5) ways to pick 5 women. By the rule of product, there are C(85,5)*C(15,5) ways to pick the committee of 5 men and 5 women.