CS 534 Spring 2013. CS Dept. WPI. HW 4 – Problem 2. Solutions by Andrew Sawchuk

Problem 2.

(¬C) ¬ [ [∀i [Mushroom(i) ⇒ ¬ White(i)]] ⇒ [¬∃j [Edible (j) ∧ Organic(j)]] ]

 ¬ [ [∀i [¬ Mushroom(i) ∨ ¬ White(i)]] ⇒ [¬∃j [Edible (j) ∧ Organic(j)]] ]

 ¬ [ ¬ [∀i [¬ Mushroom(i) ∨ ¬ White(i)]] ∨ [¬∃j [Edible (j) ∧ Organic(j)]] ]

 [∀i [¬ Mushroom(i) ∨ ¬ White(i)]] ∧ ¬ [¬∃j [Edible (j) ∧ Organic(j)]]

 At this point, break the 2 clauses apart into C.1 and C.2:

(¬C.1) ∀i [¬ Mushroom(i) ∨ ¬ White(i)]

 **¬ Mushroom(i) ∨ ¬ White(i)**

(¬C.2) ¬ [¬∃j [Edible (j) ∧ Organic(j)]]

 ∃j [Edible (j) ∧ Organic(j)]

 Edible (A) ∧ Organic(A), where A is a new constant symbol

(¬C.2.1) **Edible (A)**

(¬C.2.2) **Organic(A)**

(A1) ∀x [Edible(x) ∨ Poisonous(x) ⇒ Mushroom(x)]

 ∀x [¬ [Edible(x) ∨ Poisonous(x)] ∨ Mushroom(x)]

 ∀x [[¬ Edible(x) ∧ ¬ Poisonous(x)] ∨ Mushroom(x)]

 [¬ Edible(x) ∧ ¬ Poisonous(x)] ∨ Mushroom(x)

 [¬ Edible(x) ∨ Mushroom(x)] ∧ [¬ Poisonous(x) ∨ Mushroom(x)]

(A1.1) **¬ Edible(x) ∨ Mushroom(x)**

(A1.2) **¬ Poisonous(x) ∨ Mushroom(x)**

(A2) ∀y [Mushroom(y) ⇒ Grey(y) ∨ White(y) ∨ Yellow(y)]

 ∀y [¬ Mushroom(y) ∨ Grey(y) ∨ White(y) ∨ Yellow(y)]

**¬ Mushroom(y) ∨ Grey(y) ∨ White(y) ∨ Yellow(y)**

(A3) ∀z [Edible(z) ⇒ ¬ Yellow(z)]

 ∀z [¬ Edible(z) ∨ ¬ Yellow(z)]

 **¬ Edible(z) ∨ ¬ Yellow(z)**

(A4) ∀w [Mushroom(w) ∧ Organic(w) ⇒ ¬ Grey(w)]

 ∀w [¬ [Mushroom(w) ∧ Organic(w)] ∨ ¬ Grey(w)]

 ∀w [ [¬ Mushroom(w) ∨ ¬ Organic(w)] ∨ ¬ Grey(w)]

 **¬ Mushroom(w) ∨ ¬ Organic(w) ∨ ¬ Grey(w)**

So, in summary, we have:

(¬C.1) **¬ Mushroom(i) ∨ ¬ White(i)**

(¬C.2.1) **Edible (A)**

(¬C.2.2) **Organic(A)**

(A1.1) **¬ Edible(x) ∨ Mushroom(x)**

(A1.2) **¬ Poisonous(x) ∨ Mushroom(x)**

(A2) **¬ Mushroom(y) ∨ Grey(y) ∨ White(y) ∨ Yellow(y)**

(A3) **¬ Edible(z) ∨ ¬ Yellow(z)**

(A4) **¬ Mushroom(w) ∨ ¬ Organic(w) ∨ ¬ Grey(w)**

Resolution steps:

(¬C.1) ¬ Mushroom(x) ∨ ¬ White(x) rename i as x

(A1.1) ¬ Edible(x) ∨ Mushroom(x)

(R1) **¬ Edible(x) ∨ ¬ White(x)**

(R1) ¬ Edible(y) ∨ ¬ White(y) rename x as y

(A2) ¬ Mushroom(y) ∨ Grey(y) ∨ White(y) ∨ Yellow(y)

(R2) **¬ Edible(y) ∨¬ Mushroom(y) ∨ Grey(y) ∨ Yellow(y)**

(R2) ¬ Edible(z) ∨¬ Mushroom(z) ∨ Grey(z) ∨ Yellow(z) rename y as z

(A3) ¬ Edible(z) ∨ ¬ Yellow(z)

(R3)  **¬ Edible(z) ∨¬ Mushroom(z) ∨ Grey(z)**

(R3) ¬ Edible(x) ∨ ¬ Mushroom(x) ∨ Grey(x) rename z as x

(A1.1) ¬ Edible(x) ∨ Mushroom(x)

(R4) **¬ Edible(x) ∨ Grey(x)**

(R4) ¬ Edible(w) ∨ Grey(w) rename x as w

(A4) ¬ Mushroom(w) ∨ ¬ Organic(w) ∨ ¬ Grey(w)

(R5)  **¬ Edible(w) ∨ ¬ Mushroom(w) ∨ ¬ Organic(w)**

(R5) ¬ Edible(w) ∨ ¬ Mushroom(w) ∨ ¬ Organic(w)

(A1.1) ¬ Edible(w) ∨ Mushroom(w) rename x as w

(R6)  **¬ Edible(w) ∨ ¬ Organic(w)**

(R6) ¬ Edible(A) ∨ ¬ Organic(A) substitute A for w

(¬C.2.1) Edible (A)

(R7) ¬ Organic(A)

(R7) ¬ Organic(A)

(¬C.2.2) Organic(A)

(R8) { }

Empty set achieved, proof by refutation holds, and therefore C is entailed by the conjunction of A1, A2, A3, and A4.