CS 534 Spring 2013. CS Dept. WPI. HW 4 – Problem 1. Solutions by Andrew Sawchuk

Problem 1.

1. (A1.2) ¬ Breezy([1,1]) ⇔ ¬ Pit([1,2]) ∧ ¬ Pit([2,1])

 (A1.3) ¬ Breezy([1,2]) ⇔ ¬ Pit([1,1]) ∧ ¬ Pit([2,2])

 (A1.4) Breezy([2,1]) ⇔ Pit([1,1]) ∨ Pit([3,1]) ∨ Pit([2,2])

2. (A1.2.1) ¬ Breezy([1,1]) ⇒ ¬ Pit([1,2]) ∧ ¬ Pit([2,1])

 Breezy([1,1]) ∨ (¬ Pit([1,2]) ∧ ¬ Pit([2,1]))

 (Breezy([1,1]) ∨ ¬ Pit([1,2])) ∧ (Breezy([1,1]) ∨ ¬ Pit([2,1]))

(A1.2.1.1) **Breezy([1,1]) ∨ ¬ Pit([1,2])**

(A1.2.1.2) **Breezy([1,1]) ∨ ¬ Pit([2,1])**

 (A1.2.2) ¬ Pit([1,2]) ∧ ¬ Pit([2,1]) ⇒ ¬ Breezy([1,1])

 ¬ (¬ Pit([1,2]) ∧ ¬ Pit([2,1])) ∨ ¬ Breezy([1,1])

 **Pit([1,2]) ∨ Pit([2,1]) ∨ ¬ Breezy([1,1])**

 (A1.3.1) ¬ Breezy([1,2]) ⇒ ¬ Pit([1,1]) ∧ ¬ Pit([2,2])

 Breezy([1,2]) ∨ (¬ Pit([1,1]) ∧ ¬ Pit([2,2]))

 (Breezy([1,2]) ∨ ¬ Pit([1,1])) ∧ (Breezy([1,2]) ∨ ¬ Pit([2,2]))

(A1.3.1.1) **Breezy([1,2]) ∨ ¬ Pit([1,1])**

(A1.3.1.2) **Breezy([1,2]) ∨ ¬ Pit([2,2])**

 (A1.3.2) ¬ Pit([1,1]) ∧ ¬ Pit([2,2]) ⇒ ¬ Breezy([1,2])

 ¬ (¬ Pit([1,1]) ∧ ¬ Pit([2,2])) ∨ ¬ Breezy([1,2])

**Pit([1,1]) ∨ Pit([2,2]) ∨ ¬ Breezy([1,2])**

 (A1.4.1) Breezy([2,1]) ⇒ Pit([1,1]) ∨ Pit([3,1]) ∨ Pit([2,2])

  **¬ Breezy([2,1]) ∨ Pit([1,1]) ∨ Pit([3,1]) ∨ Pit([2,2])**

 (A1.4.2) Pit([1,1]) ∨ Pit([3,1]) ∨ Pit([2,2]) ⇒ Breezy([2,1])

 ¬ (Pit([1,1]) ∨ Pit([3,1]) ∨ Pit([2,2])) ∨ Breezy([2,1])

 (¬ Pit([1,1]) ∧ ¬ Pit([3,1]) ∧ ¬ Pit([2,2])) ∨ Breezy([2,1])

 (¬ Pit([1,1]) ∨ Breezy([2,1])) ∧

 (¬ Pit([3,1]) ∨ Breezy([2,1])) ∧

 (¬ Pit([2,2]) ∨ Breezy([2,1]))

 (A1.4.2.1) **¬ Pit([1,1]) ∨ Breezy([2,1])**

(A1.4.2.2) **¬ Pit([3,1]) ∨ Breezy([2,1])**

 (A1.4.2.3) **¬ Pit([2,2]) ∨ Breezy([2,1])**

(K1) **¬ Breezy([1,1])**

(K2) **¬ Breezy([1,2])**

(K3) **Breezy([2,1])**

(¬C) **¬ Pit([3,1])**

So, in summary:

(A1.2.1.1) **Breezy([1,1]) ∨ ¬ Pit([1,2])**

(A1.2.1.2) **Breezy([1,1]) ∨ ¬ Pit([2,1])**

 (A1.2.2)  **Pit([1,2]) ∨ Pit([2,1]) ∨ ¬ Breezy([1,1])**

(A1.3.1.1) **Breezy([1,2]) ∨ ¬ Pit([1,1])**

(A1.3.1.2) **Breezy([1,2]) ∨ ¬ Pit([2,2])**

 (A1.3.2) **Pit([1,1]) ∨ Pit([2,2]) ∨ ¬ Breezy([1,2])**

 (A1.4.1) **¬ Breezy([2,1]) ∨ Pit([1,1]) ∨ Pit([3,1]) ∨ Pit([2,2])**

 (A1.4.2.1) **¬ Pit([1,1]) ∨ Breezy([2,1])**

(A1.4.2.2) **¬ Pit([3,1]) ∨ Breezy([2,1])**

 (A1.4.2.3) **¬ Pit([2,2]) ∨ Breezy([2,1])**

(K1) **¬ Breezy([1,1])**

(K2) **¬ Breezy([1,2])**

(K3) **Breezy([2,1])**

(¬C) **¬ Pit([3,1])**

Resolution steps:

 (A1.4.1) ¬ Breezy([2,1]) ∨ Pit([1,1]) ∨ Pit([3,1]) ∨ Pit([2,2])

(¬C) ¬ Pit([3,1])

 (R1) **¬ Breezy([2,1]) ∨ Pit([1,1]) ∨ Pit([2,2])**

 (R1) ¬ Breezy([2,1]) ∨ Pit([1,1]) ∨ Pit([2,2])

(A1.3.1.1) Breezy([1,2]) ∨ ¬ Pit([1,1])

 (R2) **¬ Breezy([2,1]) ∨ Pit([2,2]) ∨ Breezy([1,2])**

 (R2) ¬ Breezy([2,1]) ∨ Pit([2,2]) ∨ Breezy([1,2])

(K3) Breezy([2,1])

 (R3) **Pit([2,2]) ∨ Breezy([1,2])**

 (R3) Pit([2,2]) ∨ Breezy([1,2])

(A1.3.1.2) Breezy([1,2]) ∨ ¬ Pit([2,2])

 (R4) **Breezy([1,2])**

 (R4) Breezy([1,2])

(K2) ¬ Breezy([1,2])

 (R5) **{ }**

 Empty set achieved, proof by refutation holds, and therefore C is entailed by our Knowledge Base.