Transmission Errors Error Detection and Correction



Advanced Computer Networks C13

Transmission Errors Outline

- Error Detection versus Error Correction
 Hamming Distances and Codes
- Linear Codes Parity
- . Internet Checksum
- Polynomial Codes
- Cyclic Redundancy Checking (CRC)
- Properties for Detecting Errors with Generating Polynomials



Transmission Errors

- . Transmission errors are caused by:
 - thermal noise {Shannon}
 - impulse noise (e..g, arcing relays)
 - signal distortion during transmission (attenuation)
 - crosstalk
 - voice amplitude signal compression (companding)
 - quantization noise (PCM)
 - jitter (variations in signal timings)
 - receiver and transmitter out of synch.



Error Detection and Correction

- error detection :: adding enough "extra" bits (redundancy) to deduce that there is an error but not enough bits to correct the error.
- If only error detection is employed in a network transmission → a retransmission is necessary to recover the frame (data link layer) or the packet (network layer).
- At the data link layer, this is referred to as ARQ (Automatic Repeat reQuest).



Error Detection and Correction

- error correction :: requires enough additional redundant bits to deduce what the correct bits must have been.
- Examples
- Hamming Codes
- FEC = Forward Error Correction found in MPEG-4 for streaming multimedia.



Hamming Codes

codeword :: a legal dataword consisting of m data bits and r redundant bits.

Error detection involves determining if the received message matches one of the legal codewords.

Hamming distance :: the number of bit positions in which two bit patterns differ.
Starting with a complete list of legal codewords, we need to find the two codewords whose Hamming distance is the smallest. This determines the Hamming distance of the code.



Error Correcting Codes



Order of bit transmission

Figure 3-7. Use of a Hamming code to correct burst errors.

Tanenbaum



Hamming Distance

(a) A code with poor distance properties

(b) A code with good distance properties

X

0

0

0

X

X

0

0

0

0

0

X

0

X



x = codewords

o = **non-codewords**



Hamming Codes

- To detect d single bit errors, you need a d+1 code distance.
- To correct d single bit errors, you need a 2d+1 code distance.
- →In general, the price for redundant bits is too expensive to do error correction for network messages.

Network protocols normally use error detection and ARQ.



Error Detection

- Note Errors in network transmissions are bursty.
- \rightarrow The percentage of damage due to errors is lower.
- \rightarrow It is harder to detect and correct network errors.
- Linear codes
 - Single parity check code :: take k information bits and appends a single check bit to form a codeword.
 - Two-dimensional parity checks
- . IP Checksum
- Polynomial Codes

Example: CRC (Cyclic Redundancy Checking)



General Error Detection System





Error Detection System Using Check Bits





Two-dimensional Parity Check Code

1	0	0	1	0	0	
0	1	0	0	0	1	
1	0	0	1	0	0	
1	1	0	1	1	0	
1	0	0	1	1	1	-

Last column consists of check bits for each row

Bottom row consists of check bit for each column



Multiple Errors





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Internet Checksum

```
unsigned short cksum(unsigned short *addr, int count)
       /*Compute Internet Checksum for "count" bytes
        * beginning at location "addr".
       */
   register long sum = 0;
   while ( count > 1 ) {
       /* This is the inner loop*/
             sum += *addr++;
            count -=2;
       }
       /* Add left-over byte, if any */
   if (\text{count} > 0)
       sum += *addr;
       /* Fold 32-bit sum to 16 bits */
   while (sum >>16)
       sum = (sum \& 0xfff) + (sum >> 16);
   return ~sum;
}
```



Polynomial Codes

- Used extensively.
- Implemented using shift-register circuits for speed advantages.
- Also called CRC (cyclic redundancy checking) because these codes generate check bits.
- Polynomial codes :: bit strings are treated as representations of polynomials with ONLY binary coefficients (0's and 1's).



Polynomial Codes

The *k bits* of a message are regarded as the coefficient list for an information polynomial of degree k-1.

$$\mathbf{I} :: i(x) = i_{k-1} x^{k-1} + i_{k-2} x^{k-2} + \dots + i_{k-1} x + i_{k-2}$$

Example:

 $i(x) = x^6 + x^4 + x^3$

1011000



Polynomial Notation

- Encoding process takes i(x) produces a codeword polynomial b(x) that contains information bits and additional check bits that satisfy a pattern.
- Let the codeword have n bits with k information bits and n-k check bits.
- We need a generator polynomial of degree
 n-k of the form



Note - the first and last coefficient are always 1.









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Polynomial Arithmetic





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CRC Algorithm

CRC Steps:

1) Multiply *i(x)* by x^{n-k} (puts zeros in (n-k) low order positions) quotient remainder

 $x^{n-k}i(x) = g(x) q(x) + r(x)$

- 2) Divide x^{n-k} i(x) by g(x)
- 3) Add remainder r(x) to x^{n-k} i(x)(puts check bits in the *n-k* low order positions): $b(x) = x^{n-k}i(x) + r(x)$ \leftarrow transmitted codeword



CRC Example

Information: $(1,1,0,0) \implies i(x) = x^3 + x^2$ Generator polynomial: $g(x) = x^3 + x + 1$ Encoding: $x^3i(x) = x^6 + x^5$					
$x^3 + x^2 + x$	1110				
$x^{3} + x + 1$) $x^{6} + x^{5}$ $x^{6} + x^{4} + x^{3}$	1011) 1100000 1011				
$x^5 + x^4 + x^3$ $x^5 + x^3 + x^2$	1110 1011				
$\begin{array}{c} X^4 + X^2 \\ X^4 + X^2 + X \end{array}$	1010 1011				
Transmitted codeword: $b(x) = x^{6} + x^{5} + x$ $\implies \underline{b} = (1,1,0,0,0,1,0)$	010 Leon-Garcia & Widjaja: <i>Communication Networks</i>				



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CRC Long Division



Figure 2.15 CRC Calculation using Polynomial Long Division

P&D



Generator Polynomial Properties for Detecting Errors

GOAL :: minimize the occurrence of an error going undetected.

Undetected means:

E(x) / G(x) has no remainder.



GP Properties for Detecting Errors

1. Single bit errors: $e(x) = x^i$ $0 \le i \le n-1$

If g(x) has more than one non-zero term, it cannot divide e(x)

2. Double bit errors: $e(x) = x^i + x^j$ $0 \le i < j \le n-1$ = $x^i (1 + x^{j-i})$

If g(x) is primitive polynomial, it will not divide $(1 + x^{j-i})$ for $j-i \le 2^{n-k} \le 1$

3. Odd number of bit errors: e(1) = 1
If number of errors is odd.
If g(x) has (x+1) as a factor, then g(1) = 0 and all codewords have an even number of 1s.



GP Properties for Detecting Errors



 $e(x) = x^{f} d(x)$ where deg(d(x)) = L-1 g(x) has degree n-k; g(x) cannot divide d(x) if deg(g(x))> deg(d(x))

if L = (n-k) or less: all will be detected if L = (n-k+1): deg(d(x)) = deg(g(x))i.e. d(x) = g(x) is the only undetectable error pattern, fraction of bursts which are undetectable = $1/2^{L-2}$ if L > (n-k+1): fraction of bursts which are undetectable = $1/2^{n-k}$

Standard Generating Polynomials

Six generator polynomials that have become international standards are:

 $CRC-8 = x^8 + x^2 + x + 1$ $CRC-10 = x^{10}+x^9+x^5+x^4+x+1$ $CRC-12 = x^{12}+x^{11}+x^3+x^2+x+1$ $CRC-16 = x^{16} + x^{15} + x^2 + 1$ $CRC - CCITT = x^{16} + x^{12} + x^5 + 1$ $CRC-32 = x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11}$ $+x^{10}+x^{8}+x^{7}+x^{5}+x^{4}+x^{2}+x+1$ IEEE link-level protocol standard {in hardware adaptor}



Basic ARQ with CRC





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