Expanding the Model-Tracing Architecture: A 3rd Generation Intelligent Tutor for Algebra Symbolization

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Abstract. Following Computer Aided Instruction systems, 2nd generation tutors are Model-Tracing Tutors (MTTs) (Anderson & Pelletier, 1991) which are intelligent tutoring systems that have been very successful at aiding student learning, but have not reached the level of performance of experienced human tutors (Anderson et al., 1995). To that end, this paper presents a new architecture called ATM ("Adding a Tutorial Model"), which is an extension to model-tracing, that allows these tutors to engage in a dialog that is more like what experienced human tutors do. Specifically, while MTTs provide *hints* toward *doing* the next problem-solving step, this 3rd generation of tutors, the ATM architecture, adds the capability to *ask* questions towards *thinking* about the knowledge behind the next problem-solving step. We present a new tutor built in ATM, called *Ms. Lindquist*, which is designed to carry on a tutorial dialog about algebra symbolization. The difference between ATM and MTT is the separate *tutorial model* that encodes *pedagogical content knowledge* in the form of different tutorial strategies, which were partially developed by observing an experienced human tutor. Ms. Lindquist has tutored thousands of students at www.AlgebraTutor.org. Future work will reveal if Ms. Lindquist is a better tutor because of the addition of the tutorial model.

Keywords. Intelligent tutoring systems, teaching strategies, model-tracing, student learning, algebra.

INTRODUCTION

This paper describes a step toward the next generation of practical intelligent tutoring systems. Let us assume that CAI (Computer Aided Instruction) systems were 1st generation tutors (see Kulik, Bangert & Williams, 1983). They presented a page of text or graphics and, depending upon the student's answer, presented a different page. The 2nd generation of tutors, Model-Tracing Tutors (MTTs) (Anderson & Pelletier, 1991), allow the tutor to follow the problem-solving steps of the student through the use of a detailed cognitive model of the domain. MTTs have had considerable success (Koedinger, Anderson, Hadley & Mark, 1997; Anderson, Corbett, Koedinger & Pelletier, 1995; Shelby et al., 2001) in improving student learning. MTTs have also had commercial success with more than 1% of American high schools now using MTTs sold by Carnegie Learning Incorporated (www.CarnegieLearning.com).

Despite the success of MTTs, they have not reached the level of performance of experienced human tutors (Anderson et al., 1995; Bloom, 1984) and instruct in ways that are

quite different from human tutors (Moore, 1996). Various researchers have criticized model-tracing (Ohlsson, 1986; McArthur, Stasz, & Zmuidzinas, 1990). For instance, McArthur et al. (1990) criticized Anderson's et al. (1985) model-tracing ITS and model-tracing in general "because each incorrect rule is paired with a particular tutorial action (typically a stored message)…Anderson's tutor is tactical, driven by local student errors (p. 200)." They go on to argue for the need for a strategic tutor. The mission of the Center for Interdisciplinary Research on Constructive Learning Environments (CIRCLE) is 1) to study human tutoring and 2) to build and test a new generation of tutoring systems that encourage students to construct the target knowledge instead of telling it to them (VanLehn et al., 1998). The hypothesis that underlies this research area is that we can improve computer tutors (i.e., improve the learning of students who use them) by making them more like experienced human tutors. Ideally, the best human tutors should be chosen to model, but it is difficult to determine which are the best. This particular study is limited in that it is based upon a single experienced tutor. A more specific assumption of this work is that students will learn better if they are engaged in a *dialog* to help them construct knowledge for themselves, rather than just being *hinted* toward inducing the knowledge from problem-solving experiences.

This paper is also focused on a particular aspect of tutoring. In particular, it is focused on what we call the *knowledge-search loop*. We view a tutoring session as containing several loops. The outermost loop is the *curriculum loop*, which involves determining the next best problem to work on. Inside of this loop, there is the *problem-solving loop*, which involves helping the student select actions in the problem-solving process (e.g., the next equation to write down, or the next element to add to a free-body diagram in a physics problem). Traditional model-tracing is focused at this level, and is effective because it can follow the individual path of a student's problem-solving through a complicated problem-solving process. However, if the student is stuck, it can only provide hints or rhetorical questions toward what the student should do next. Model-tracing tutors do not ask new questions that might help students towards identifying or constructing relevant knowledge. In contrast, a human tutor might "dive down" into what we call the *knowledge-search loop.* Aiding students in knowledge search involves asking the student questions whose answers are not necessarily part of the problem-solving process, but are chosen to assist the student in learning the knowledge needed at the problem-solving level. It is this innermost *knowledge-search* loop that this paper is focused upon because is it has been shown that most learning happens only when students reach an impasse (VanLehn, Siler, Murray, Yamauchi & Baggett, 2003). In addition, VanLehn et al. suggested that different types of tutorial strategies were needed for different types of impasses.

The power of the model-tracing architecture has been in its simplicity. It has been possible to build practical systems with this architecture, while capturing some, but not all, features of effective one-on-one tutoring. This paper presents a new architecture for building such systems called ATM (for **A**dding a **T**utorial **M**odel) (Heffernan, 2001). ATM is intended to go a step further but maintain simplicity so that practical systems can be built. ATM incorporates more features of effective tutoring than model-tracing tutors, but does not aspire to incorporate all such features.

A number of 3rd generation systems have been developed (Core, Moore & Zinn, 2000; VanLehn et al., 2000; Graesser et al., 1999; Aleven & Koedinger, 2000a). In order to concretely illustrate the ATM architecture, this paper also presents an example of a tutor built within this architecture, called *Ms. Lindquist*. Ms. Lindquist is not only able to model-trace student actions, but can be more human-like in carrying on a running conversation with the student, complete with probing questions, positive and negative feedback, follow-up questions in embedded sub-

dialogs, and requests for explanations as to why something is correct. In order to build Ms. Lindquist we have expanded the model-tracing paradigm so that Ms. Lindquist not only has a model of the student, but also has a model of tutorial reasoning. Building a tutorial model is not a new idea, (e.g., Clancey, 1982), but incorporating it into the model-tracing architecture is new. Traditional model-tracing tutors have an implicit model of the tutor; that model is that tutors keep students on track by giving (sometimes implicitly) positive feedback as well as making comments on student's wrong actions. Traditional model-tracing tutors do not allow tutors to ask new question to break steps down, nor do they allow multi-step lines of questioning. Based on observation of both an experienced tutor and cognitive research (Heffernan & Koedinger, 1997, 1998), this tutorial model has multiple tutorial strategies at its disposal.

MTTs are successful because they include a detailed model of how students solve problems. The ATM architecture expands the MTT architecture by also including a model of what experienced human tutors do when tutoring. Specifically, similar to the model of the student, we include a tutorial model that captures the knowledge that a tutor needs to be a good tutor for the particular domain. For instance, some errors indicate minor slips while others will indicate major conceptual errors. In the first case, the tutor will just respond with a simple corrective getting the student back on track (which is what model-tracing tutors do well), but in the second case, a good tutor will tend to respond with a more extended dialog (something that is impossible in the traditional model-tracing architecture).

We believe a good human tutor needs at least three types of knowledge. First, they need to know the domain that they are tutoring, which is what traditional MTTs emphasize by being built around a model of the domain. Secondly, they need general pedagogical knowledge about how to tutor. Thirdly, good tutors need what Shulman (1986) calls *pedagogical content knowledge*, which is the knowledge at the intersection of domain knowledge and general pedagogical knowledge. A tutor's "pedagogical content knowledge" is the knowledge that he or she has about how to teach a specific skill or content domain, like algebra. A good tutor is not simply one who knows the domain, nor is a good tutor simply one who knows general tutoring rules. A good tutor is one who also has content specific strategies (an example will be given later in the section "The Behavior of an Experienced Human Tutor") that can help a student overcome common difficulties. McArthur et al. (1990) recognized the need to model the strategies used by experienced human tutors, and that such a model could be a component of an intelligent tutoring system.

Building a traditional model-tracing tutor is not easy, and unfortunately, the ATM architecture involves only additional work. Authoring in Anderson & Pelletier's (1991) model-tracing architecture involves significant work. Programming is needed to implement a cognitive model of the domain, and ideally, this model involves psychological research to determine how students actually solve problems in that domain (e.g., Heffernan & Koedinger, 1997; Heffernan & Koedinger, 1998). The ATM architecture involves the additional work of first analyzing the tutorial strategies used by experienced human tutors and then implementing such strategies in a tutorial model. This step should be done before building a cognitive model, as it constrains the nature and level of detail in the cognitive model that is needed to support the tutorial model's selection of tutorial options.

In this paper, we first describe the model-tracing architecture used to build second-generation systems and then present an example of a tutor built in that architecture. Then we present an analysis of an experienced human tutor that serves as a basis for the design on Ms. Lindquist and the underlying ATM architecture. We illustrate the ATM architecture by describing how the Ms. Lindquist tutor was constructed within. The Ms. Lindquist tutor included both a model of the student (the research that went into the student model is described in Heffernan & Koedinger, 1997 & 1998) as well as a model of the tutor.

THE SECOND GENERATION ARCHITECHTURE: MODEL-TRACING

The Model-Tracing Architecture was invented by researchers at Carnegie Mellon University (Anderson & Pelletier, 1991; Anderson, Boyle & Reiser, 1985) and has been extensively used to build tutors, some of which are now sold by Carnegie Learning, Inc. (Corbett, Koedinger, Hadley, 2001). These tutors have been used by thousands of schools across the country and have been proven to be very successful (Koedinger, Anderson, Hadley & Mark, 1995). Each tutor is constructed around a cognitive model of the problem-solving knowledge students are acquiring. The model reflects the ACT-R theory of skill knowledge (Anderson, 1993) in assuming that problem-solving skills can be modeled as a set of independent production rules. Production rules are if-then rules that represent different pieces of knowledge (A concrete example of a production will be given in the section on "Ms. Lindquist's Cognitive Student Model".) Model-tracing provides a particular approach to implementing the standard components of an intelligent tutoring system, which typically include a graphical user-interface, expert model, student model and pedagogical model. Of these components, MTTs emphasize the first three.

Anderson, Corbett, Koedinger & Pelletier (1995) claim that the first step in building a MTT is to define the interface in which the problem-solving will occur. The interface is usually analogous to what the student would do on a piece of paper to solve the problem. The interface enables students to reify steps in their problem-solving performance, thus enabling the computer to be able to follow the problem-solving steps the student is using.

The main idea behind the model-tracing architecture, is that if a model of what the student might do exists (i.e., a cognitive model including different correct and incorrect steps that the student could take) then a system will be able to offer appropriate feedback to students including positive feedback and hints to the student if they are in need of help. Each task that a student is presented with can be solved by applying different pieces of knowledge. Each piece of knowledge is represented by a production rule. The expert model contains the complete set of productions needed to solve the problems, as well as the "buggy" productions. Each buggy production represents a commonly occurring incorrect step. The somewhat radical assumption of model-tracing tutors is that the set of productions needs to be **complete.** This requires the cognitive modeler to model all the different ways to solve a problem as well as all the different ways of producing the common errors. If the student does something that cannot be produced by the model, it is marked as wrong. The model-tracing algorithm uses the cognitive model to "model-trace" each step the student takes in a complex problem-solving search space. This allows the system to provide feedback on each problem-solving action as well as give hints if the student is stuck.

Specifically, when the student answers a question, the model-tracing algorithm is executed in an attempt to do a type of *plan recognition* (Kautz & Allen, 1986). For instance, if a student was supposed to simplify "7(2+2x) + 3x" and said "10+5x", a model tracer might respond with a buggy message of "Looks like you failed to distribute the 7 to the 2x". (The underlined text would be filled in by a template so that the message applies to all situations in which the student fails to distribute to the second term.) A model tracer is only able to do this if a bug rule had been written that is able to model that incorrect rule of forgetting to distribute to the second term. Note that model-tracing often involves firing rules that work correctly (like the rule that added the 2x +3x, as well as rules that do some things incorrectly).

An additional component of traditional model-tracing architecture is called *knowledge-tracing* which is a specific implementation of an "overlay" student model. An overlay student

model is one in which the student's knowledge is treated as a subset of the knowledge of the expert. As students work through a problem, the system keeps track of the probabilities that a student knows each production rule. These estimates are used to decide on the next best problem to present to the student. The ATM architecture makes no change to knowledge tracing.

Model-tracing tutors give three types of feedback to students: 1) *flag feedback*, 2) *buggy messages*, and 3) a *chain of hints*. Flag feedback simply indicates the correctness of the response, sometimes done by using a color (e.g., green=correct or red=wrong). A buggy message is a text message that is specific to the error the student made (examples below). If a student needs help, they can request a "Hint" to receive the first of a chain of hints that suggests things for the student to think about. If the student needs more help, they can continue to request a more specific hint until the "bottom-out" message is delivered that usually tells the student exactly what to type. Anderson & Pelletier (1991) argue for this type of architecture because they hypothesized that telling students what to do next would be more helpful than focusing on their errors. We agree that emphasizing bug-diagnosis is probably not particularly helpful, however simply "spewing" text at the student may not be the most pedagogically effective response. This point will be elaborated upon in the section describing Ms. Lindquist's architecture.

THIRD GENERATION SYSTEMS

The ATM architecture is our attempt to build a new architecture, that will extend the model-tracing architecture to allow for better dialog capabilities. Other researchers (Aleven & Koedinger, 2000a; Core, Moore & Zinn, 2000; Freedman & Evens, 2000; Graesser et al., 1999; VanLehn et al., 2000) have built 3rd generation systems but ATM is the first to take the approach of generalizing the successful model-tracing architecture to seamlessly integrate tutorial dialog. Besides drawing on the demonstrated strengths of model-tracing tutors, this approach allows us to show how model tracing is a simple instance of tutorial dialog. Aleven and Koedinger (2000a & 2000b) have built a geometry tutor in the traditional model-tracing framework but have added a requirement for students to explain some of their problem-solving steps. The system does natural language understanding of these explanations by parsing a student's answer. The system's goal is to use traditional buggy feedback to help students refine their explanations. Many of the hints and buggy messages ask new "questions", but they are only rhetorical. For instance, when the student justifies a step by saying "The angles in an isosceles triangle are equal" and the tutor responds with "Are all angles in an isosceles triangle equal?" the student doesn't get to say "No, it's just the base angles". Instead, the student is expected to modify the complete explanation to say "The *base* angles in an isosceles triangle are equal." Therefore, the system's strength appears to be its natural language understanding, while its weakness is in not having a rich dialog model that can break down the knowledge construction process through new non-rhetorical questions and multi-step plans.

Another tutoring system that does natural language understanding is Graesser's et al. (1999) system called "AutoTutor". AutoTutor is a system that has a "talking head" that is connected to a text-to-speech system. AutoTutor asks students questions about computer hardware and the student types a sentence in reply. AutoTutor uses latent semantic analysis to determine if a student's utterance is correct. That makes for a much different sort of student modeling than model-tracing tutors. The most impressive aspect of AutoTutor is its natural language understanding components. The AutoTutor developers (Graesser et al.,1999) de-emphasized dialog planning based on the claim that novice human tutors do not use sophisticated strategies, but nevertheless, can be effective. Auto-tutor does have multiple tutorial strategies (i.e., "Ask a

fill-in-the-blank question" or "Give negative feedback."), but these strategies are not multi-step plans. However, work is being done on a new "Dialogue Advancer Network" to increase the sophistication of its dialog planning.

The CIRCSIM-Tutor project (see Cho, Michael, Rovick, and Evens, 2000; Freedman & Evens, 1996) has done a great deal of research in building dialog-based intelligent tutors systems. Their tutoring system, while not a model-tracing tutor, engages the student in multi-step dialogs based upon two experienced human tutors. In CIRCSIM-Tutor, the dialog planning was done within the APE framework (Freedman, 2000). Freedman's approach, while developed independently, is quite similar to our approach for the tutorial model in that it is a production system that is focused on having a hierarchal view of the dialog.

VanLehn et al. (2000) are building a 3rd generation tutor by improving a 2nd generation model-tracing tutor (i.e., the Andes physics tutor) by appending onto to it a system (called Altas) that conducts multiple different short dialogs. The new system, called Atlas-Andes, is similar to our approach in that students are asked new questions directed at getting the student to construct knowledge for themselves rather than being told. Also similar to our approach is that VanLehn and colleagues have been guided by collecting examples from human tutoring sessions. While their goal and methodology are similar, their architecture for 3rd generation tutors is different. VanLehn et al. (2000) says that "Atlas takes over when Andes would have given its final hint. (p. 480)" indicating that the Atlas-Andes system is two systems that are loosely coupled together. When students are working in Atlas, they are, in effect, using a 1st generation tutor that poses multiple-choice questions and branches to a new question based on the response, albeit one that does employ a parser to map the student's response to one of the multiple-choice responses. Because of this architectural separation, the individual responses of students are no longer being model-traced or knowledge-traced. This separation is in contrast with the goal of seamless integration of model-tracing and dialog in ATM.

Carnegie Learning's Cognitive Algebra Tutor

We will now give an example of the sort of feedback traditional model-tracing tutors provide. We will look at Carnegie Learning Inc.'s tutor called the "Cognitive Algebra Tutor". This software teaches various skills in algebra (i.e., problem analysis, graphing and equation solving), but the skill we will focus on here is the symbolization process (i.e., where a student is asked to write an equation representing a problem situation). Symbolization is fundamental because if students cannot translate problems into the language of algebra, they will not be able to apply algebra to solve them. Symbolization is also a difficult task for students to master. One relevant window related to symbolizations is shown in Figure 1. where the student is expected to answer questions by completing a table shown (partially filled in).

In Figure 1, we see that the student has already identified names for three quantities (i.e., "hours worked", "The amount you would earn in your current job", and "the amount you would earn in the new job"), as well as having identified units (i.e., "hours", "dollars" and "dollars" respectively) as well as having chosen a variable (i.e., "h") to stand for the "hours worked" quantity.

One of the most difficult steps for students is generating the algebraic expression and Figure 1 shows a student who is currently in the middle of attempting to answer this sort of problem, as shown by the fact that that cell is highlighted. The student has typed in "100-4*h" but has not yet hit return. The correct answer is "100+4*h".

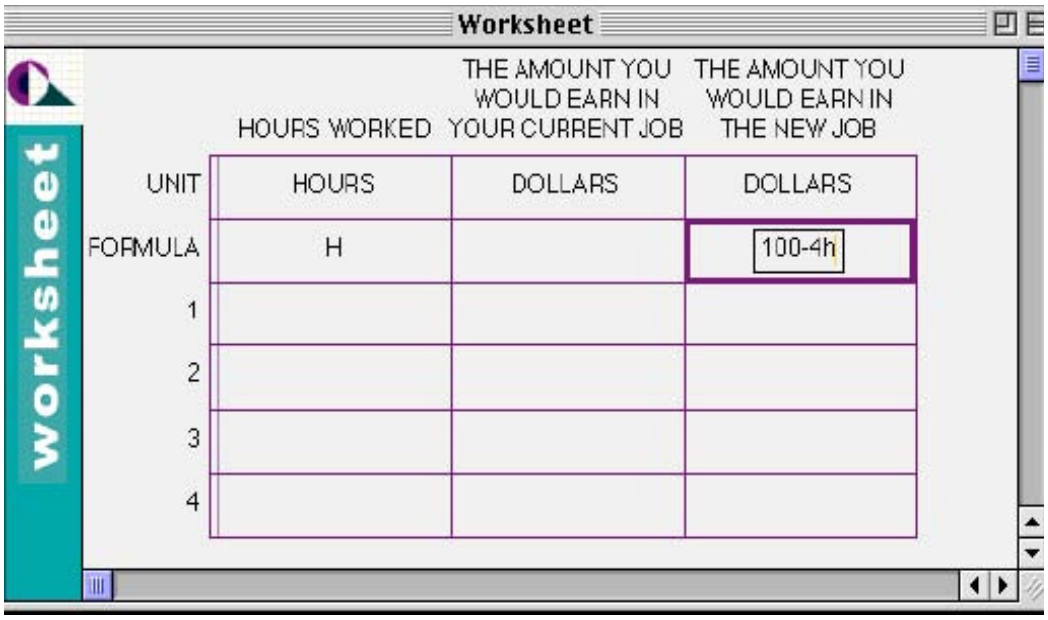

Fig. 1. The worksheet window from the Carnegie Learning tutor (circa 1999). The student has already filled in the column headings as well as the units, and is working on the formula row. The student has just entered "100-4h" but has not yet hit the return key.

Once the student hits return, the system will give flag feedback, highlighting the answer to indicate that the answer is incorrect. In addition, the model-tracing algorithm will find that this particular response can be modeled by using a buggy rule, and since there is a buggy template associated with that rule, the student is presented with the buggy message that is listed in the first row of Table 1. Table 1 also shows three other different buggy messages.

Table 1

Four different classes of errors, and associated buggy-message that are generated by Carnegie Learning's Cognitive Algebra Tutor. The third column shows a hypothetical student response, but unfortunately, the questions are only rhetorical. The ATM is meant to address this.

	Example Errors	**The buggy message generated in response to those errors**	**Possible response by the student**
1	100-4*h -4*h+100	Does the money earned in your current job increase or decrease as the number of hours worked increases?	It increases.
2	4*h 10+4*h	How many dollars do you start with when you calculate the money earned in your current job?	100 dollars
3	100+h 100+3*h	How much does the money earned in your current job change for each hour worked?	Goes up 4 dollars for every hour
4	4+100*h 100h+4	Which number should be the slope and which number should be the intercept in your formula?	The 4 dollars an hour would be the slope.

Notice how the four buggy messages are asking questions of the student that seem like very reasonable and plausible questions that a human tutor would ask a student. The last column in Table 1 shows possible responses that a student might make. Unfortunately, those are only rhetorical questions, for the student is not allowed to answer them, as such, and is only allowed to

try to answer the original question again. This is a problem the ATM architecture solves by allowing the student to be asked the question implied in this buggy message. In this hypothetical example, when the student responds "It increases" then the system can follow that question up with a question like "And 'increases' suggests what mathematical operation?" Assuming the student says "addition" the tutor can then ask "Correct. Now fix your past answer of 100-4*h". We call this collection of questions, as well as the associated responses in case of unexpected student responses, a *tutorial strategy.* The ATM architecture has been designed to allow for these sorts of tutorial strategies that require asking students new questions that foster reasoning before doing, rather than simply hinting towards what to do next.

Table 2
The list of hints provided to students upon request by the Carnegie Learning's Cognitive Algebra Tutor.

Text of Hint
Enter an expression to calculate the money earned in your current job using the hours worked.
First, consider the initial value of the money earned in your current job. Next, consider how the money earned in your current job will change for each hour.
Write an expression that means the same thing as the value of the money earned in your current job plus the change in the money earned in your current job for each hour times the hours worked.
Write an expression that means the same thing as 100+4 times the number of hours worked.
Enter 4.00H + 100.00.

Table 2 shows the hint sequence for this same symbolization question. Notice how the hints get progressively more explicit until finally the student is told what to type. One of the problems with model-tracing tutors is that sometimes students keep asking for a new hint until they get the last most specific hint (Gluck, 1999). However, maybe this is a rational strategy to use when the hints do not efficiently focus on the student's difficulty. Take a moment to consider if the chain of hints in Table 2 is likely to help the student above who just tried "100-4*h"? The 1st and 2nd hints certainly do not address this student's difficulty, and the later hints only do so very obliquely. This lack of sensitivity to the student's cognitive state is an architectural limitation that the ATM architecture is designed to overcome by creating tutors that aim to aid learning by asking the student questions which are focused on the portions that they got wrong. We call this *dynamic scaffolding* and will define this in the next section.

THE BEHAVIOR OF AN EXPERIENCED HUMAN TUTOR

We developed the ATM architecture to be able to build tutors that model the tutorial strategies that we observed in the behavior of an experienced tutor. An hour-long protocol of an experienced human tutor working with an individual student in a coached practice session was collected. The tutor was a female middle school mathematics teacher with four years of mathematics teaching experience and two years of one-on-one tutoring experience (through both University tutoring centers and through extensive private tutoring.) This tutor charged clients 40 dollars an hour. The tutor worked with one of her seventh grade students that she had not previously tutored and was given a list of problems for the student to solve. The session was

recorded on video and then transcribed using the piece of paper the student wrote his answers on. Strategies that the tutor appeared to use often and that were easy to implement were chosen to incorporate in Ms. Lindquist.

The tutoring session was quite interactive, resulting in slightly over 400 lines of transcript. The session consisted of the tutor and student working on 17 word problems. Of these 17, 7 of them were done correctly on their first attempt. The tutor did not spend much time on these correctly answered problems (consumed only 24 of the 400+ lines). The remaining ten problems represent the bulk of the lines in the protocol. Since most of the time the tutor and the student alternated speaking, it makes for an average of about 20 turns (defined as the student and then the tutor speaking) per problem. One problem took an exceptionally long time and stretched from line 17 to line 146. If this long problem is excluded the average number of turns to solve a problem would be slightly over ten turns per problem, which is still quite substantial. This finding is in agreement with the literature (Merrill et al., 1995) that suggests tutors give a great deal of feedback so that the student knows if he is right or wrong. The tutor would give immediate confirmation if an answer was correct, but if it was wrong, she seldom told the student the answer. Instead, the tutor would generally ask a targeted question thereby giving implicit negative feedback.

Table 3

An experienced tutor's transcript and Ms. Lindquist's attempt to model it

Experienced Human Tutor Exact Transcript	**The Ms. Lindquist Tutor Similar simulated input.**
240. Student: [reads the "bike-trip" problem] Cathy took a "m" mile bike ride. She rode at a speed of "s" miles per hour. She stopped for a "b" hour break. Write an expression for how long the trip took.	T1: [The "bike-trip" problem was presented] Please write an expression for the total time the trip took.
241. S: um … [Writes "s/m+b"]	S1: s/m+b
242. Tutor: How do you calculate the amount of time it takes you? If you're, if you're, if you're riding at, let's make it simple. If you are riding at 20 miles per hour, OK, and you go 100 miles, how many hours did that take you?	T2: No. Let me ask you an easier question. Please compute the time actually on the bikes if the speed was 20 miles per hour, and the distance ridden was 100 miles?
243. S: Um 5	S2: 5
244. T: 5. And how did you get that 5? How did you use the numbers 100 and ?[trails off]	T3: Correct. What math did you do to get that five?
245. S: 100 miles divided by miles per hour.	S3: 100/20
246. T: So you took the miles and divided it by the [garbled, but probably "speed"]	T4: Correct. Now write your answer of 100/20 using the variables given in the problem (i.e., put in 'm' and 's')?
247. S: Miles divided by 's' plus 'b' equals time [writes m/s+b]	S4: b+m/s
248. T: Right.	T5: Correct.

An example of the behavior of this human tutor is shown in the left column of Table 3. This example was collected and transcribed from a one-on-one tutoring session with a student working

on the “bike-trip” problem (a problem we use as one of our running examples). The right hand side of Table 3 shows a corresponding interaction with Ms. Lindquist and will be discussed later in the section on Ms. Lindquist.

The tutor in the above dialog appears to have done two things to help the student with the problem. First, the tutor focused on the problem of calculating the time actually on the bikes (i.e., the m/s part) by decomposing what was a problem with two arithmetic operators (i.e., addition and division) into a problem that had only one math operator. Presumably, this is because the student indicated he understood that the goal quantity was found by adding the amount of the break (“b”) to the time actually on the bikes. This is an example of what we call **dynamic scaffolding**, by which we mean focusing the dialog on an area where the student has had difficulty.

The second way this tutor helped the student was to apply what we call a **tutorial strategy** (similar to what McArthur et al. (1990) called *micro-plans* and what VanLehn et al. (2000) called *knowledge construction dialogs*). The particular tutorial strategy the tutor used is the one we call the *concrete articulation strategy* (called the *inductive support* strategy in Gluck, 1999, Koedinger & Anderson, 1998), which involves three steps. The first step is the *compute question* which involves asking the student to suppose one, or more, of the variables is a concrete number and then to compute a value (i.e., asking the student to calculate the time actually on bikes using 100 and 20 rather than “m” and “s”.) The second step is the *articulation question*, which asks the student to explain what math they did to arrive at that value (i.e., "How did you get that 5?"). The final step is the *generalization question,* which asks the student to generalize their answer using the variables from the problem (i.e., line 246). We observed that our experienced human tutor employed this concrete articulation strategy often (in 4 of 9 problems). We describe the strategies used by the human tutor and how they were incorporated in Ms. Lindquist in more detail in the next section.

THE ATM ARCHITECTURE

We believe that dynamic scaffolding and tutorial strategies are two pieces that current model-tracing framework does not deal with well, and thus motivate extending the model-tracing architecture by adding a separate tutorial model that can implement these new features and the ATM architecture. Figure 2 shows a side-by-side comparison of the traditional model-tracing architecture to the ATM architecture.

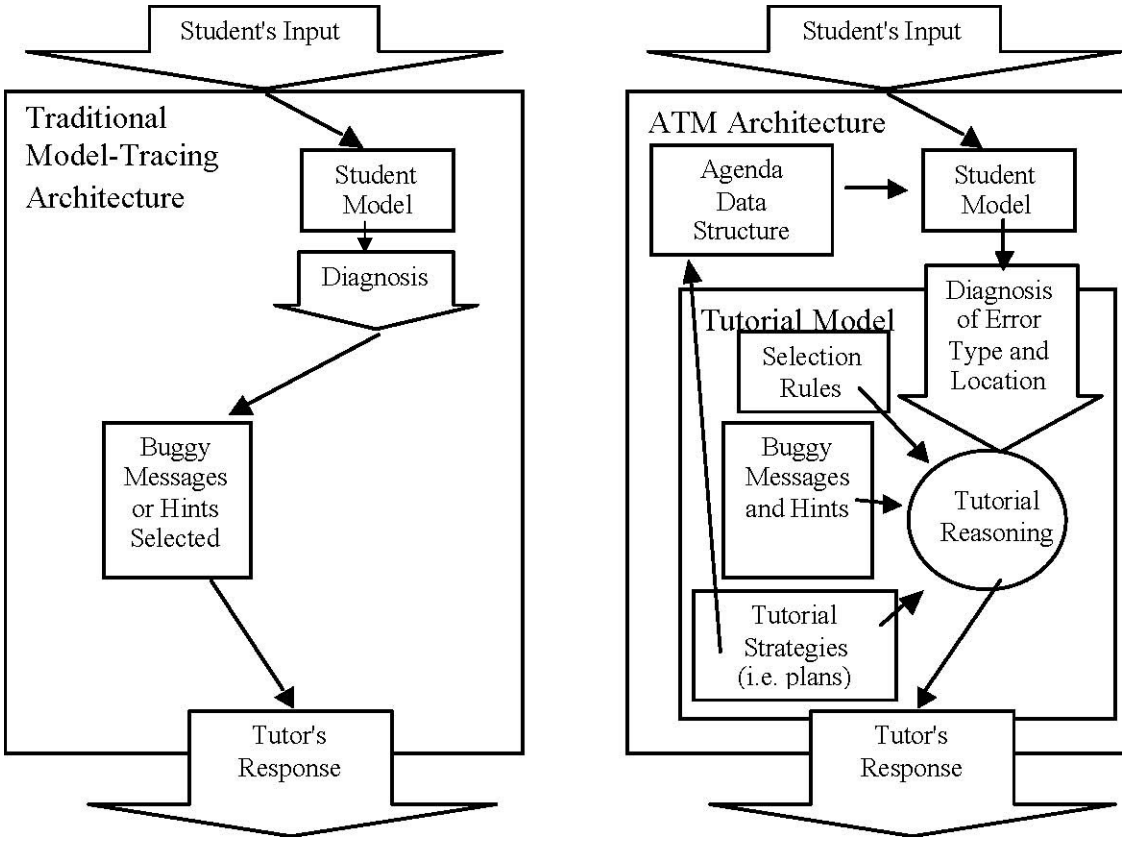

Fig. 2. A comparison of the old and the new architectures.

The traditional model-tracing architecture feeds the student's response into the model-tracing algorithm to generate a message for the student but never asks a new question, and certainly never plans out a series of follow-up questions (as we saw the experienced human tutor appear to do above with the concrete articulation strategy). A key enhancement of the ATM architecture is the agenda data structure that allows the system to keep track of the dialog history as well as the tutor's plans for follow-up questions. Once the student model has been used to diagnose any student errors, the tutorial model does the necessary reasoning to decide upon a course of action. The types of responses that are possible are to give a buggy message, give a hint or use a tutorial strategy. The *selection rules,* shown in Figure 2, are used to select between these three different types of responses. It should be noted that currently the selection rules used in Ms. Lindquist are very simple. However, selection rules can model complex knowledge, such as when to use a particular tutorial strategy for a particular student profile, or a particular student's error, or a particular context in a dialog. Research will be needed to know what constitutes **good** selection rules, so we have currently opted for simple selection rules. For instance, there is a rule that forces the system to use a tutorial strategy, when possible, as opposed to a buggy message. Another selection rule can cause the system to choose a particular tutorial strategy in response to a certain class of error.

Whereas buggy messages and hints are common between both architectures, the use of tutorial strategies triggered by selection rules makes the ATM more powerful than the traditional architecture, because the tutor is now allowed to ask new questions of the student.

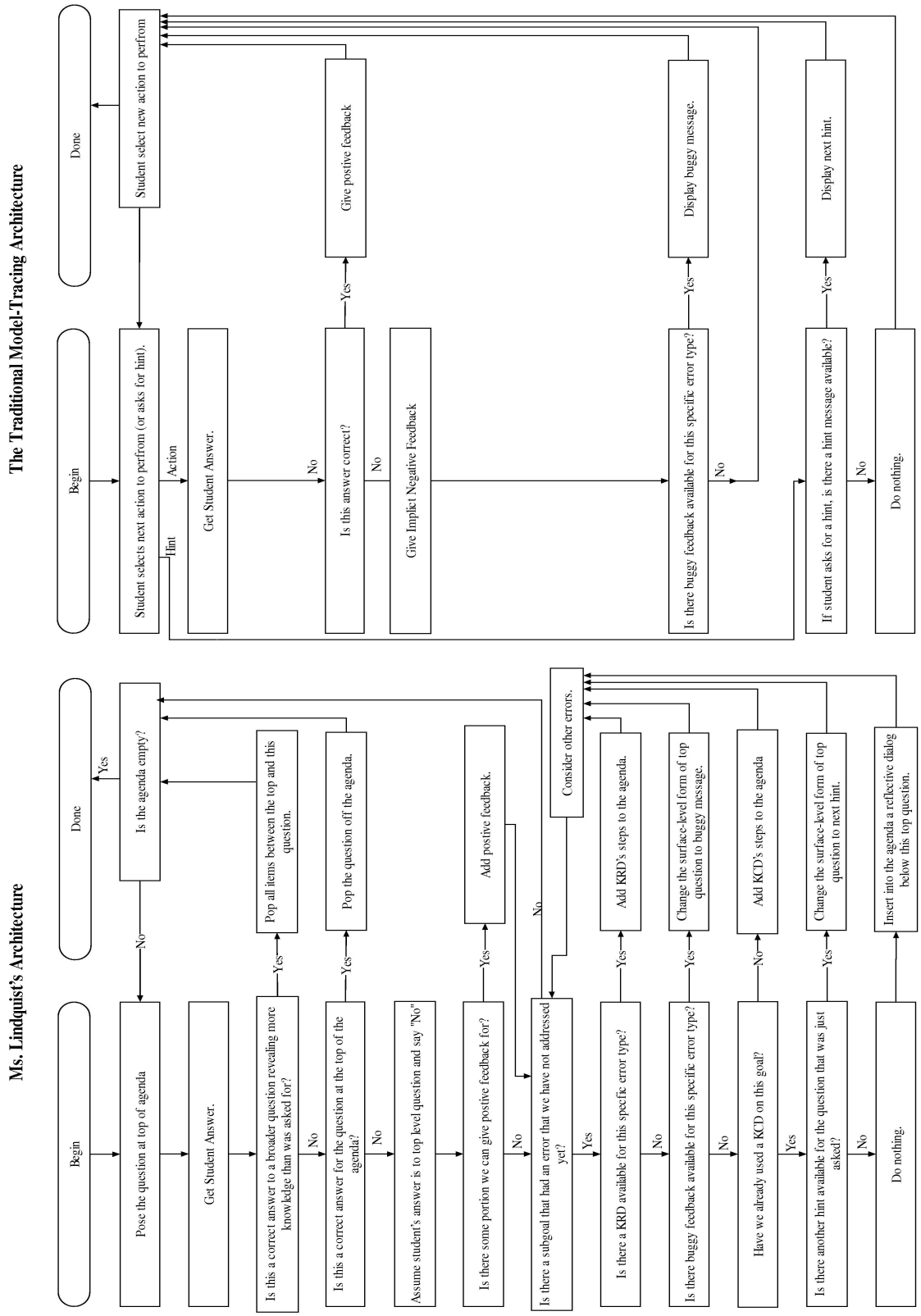

Fig. 3. Flowcharts comparing Ms Lindquist's Architecture with the traditional model-tracing architecture

The overall algorithm ATM uses is shown in Figure 3, and contrasted with traditional

model tracing tutors. The traditional model-tracing algorithm includes only buggy feedback and hints. On the other hand, the ATM architecture also includes new elements, as shown by the extra boxes in the flowchart (KCD and KRD are two types of tutorial strategies that will be discussed in the section below on “Tutorial Strategies”). The ATM architecture begins by posing the question that is at the top of the agenda structure, and waits for the student to attempt an answer. Sometimes the student's answer will reveal more information than what was asked for, as in Table 3, response S4, in which the system was expecting an answer of "m/s" but instead received an answer of "b+m/s". Strictly speaking, the student's answer of "b+m/s" is wrong for the question that was asked, however, the tutor would appear pedantic if it said "no" because "b+m/s" is an answer to a question that is lower down on the tutorial agenda. Therefore, the system treats "b+m/s" as a correct answer to the original question asking for "b+m/s". Having this mechanism in place is part of ensuring reasonable conversational coherence.

The flow diagram shows that if the student gave an answer that is correct for the question at the top of the agenda, the system pops that question off the agenda and proceeds to pose any remaining questions. However, if the student's answer is not correct, the system says "No" and then tries to add any positive feedback before entering the dynamic scaffolding subroutine. That routine tries to come up with the best plan for each error the student might have made for each subgoal. Once the system has planned a response to the first subgoal that had an error, the system will try to do the same for any remaining subgoals that have errors. The integration of model-tracing and dialog is shown in Figure 3. As Figure 3 illustrates, ATM generalizes the functionality of model-tracing (the added boxes on the right) without eliminating any of it (boxes appearing on both sides). We will now describe each of the components of the ATM architecture (Figure 2) with reference to the Ms. Lindquist tutor.

Ms. Lindquist’s Cognitive Student Model

Ms Lindquist's student model is similar to traditional student models. We used the Tertl (Anderson & Pelletier, 1991) production system, which is a simplification of the ACT (Anderson, 1993) Theory of Cognition. As mentioned above, a production system is a group of if-then rules operating on a set of what are called *working memory elements*. We use these rules to model the cognitive steps a student could use to solve a problem. Our student model has 68 production rules. Our production system can solve a problem by being given a set of working memory elements that encode, at a high level, the problem.

To make this concrete, we now provide an example. Figure 4 shows initial working memory encoding the "Anne in a lake" problem. We see that the problem has 5 quantities and two relations that link the quantities together in what we call a *quantitative network*. Our 68 productions can be broken up into several groups. Some productions are responsible for doing a search through the quantitative network to connect the givens with the goal. Other productions are used to retrieve the operator to use (e.g., +, -, *, /). Other productions are used to order the arguments (e.g., 800-40m versus 40m-800). Still other productions are used to add parentheses when needed. For example, an English version of a production that does the search:

If

You are trying to find a symbolization for an unknown quantity,
And that quantity is involved in a relation

Then

Set goals to try to symbolize the two other quantities connected to that relation,

And set a goal to retrieve the operator to use.

For example, in conjunction with the working memory elements shown in Figure 4, this production could be used to symbolize "the distance Anne has left to row" by setting goals to symbolize 1) "the distance she started from the dock" and 2) "the distance rowed so far", as well as setting a goal to retrieve the correct operator to use.

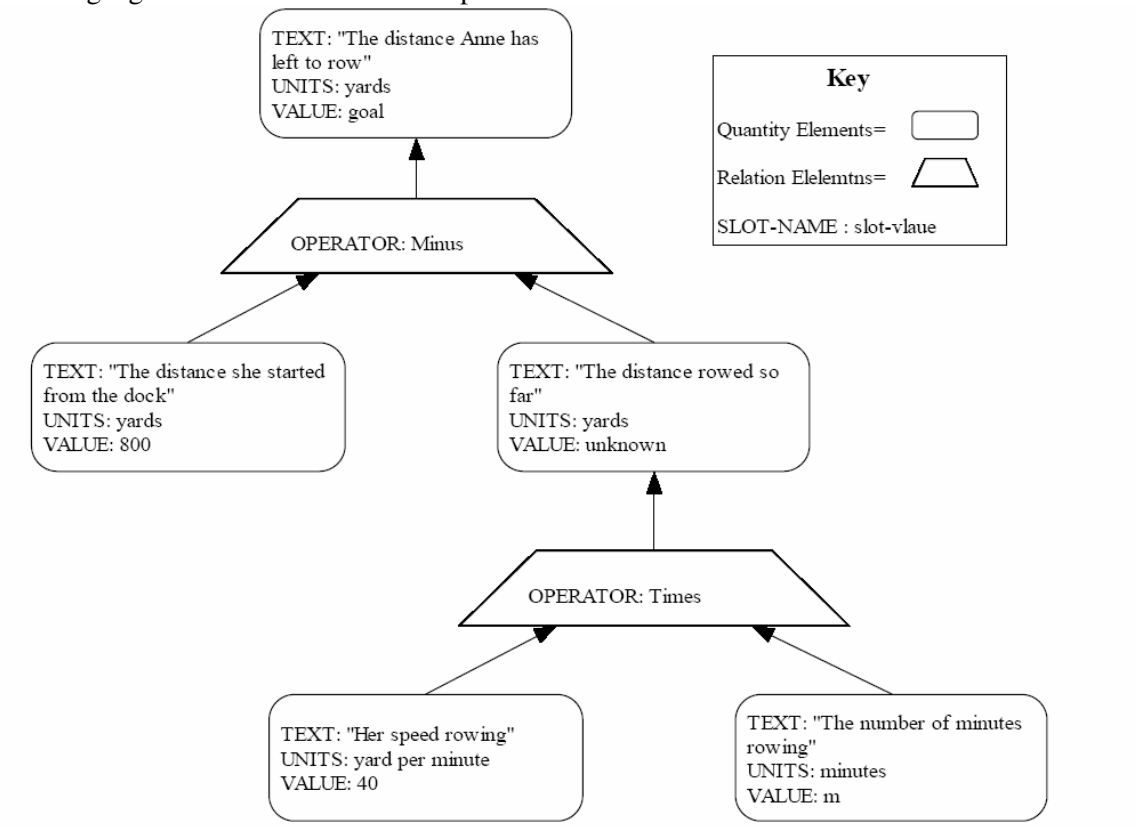

Fig. 4. The initial working memory elements for the following problem:
Ann is in a rowboat in a lake. She is 800 yards from the dock. She then rows for "m" minutes back towards the dock. Ann rows at a speed of 40 yards per minute. Write an expression for Ann's distance from the dock. Answer=800-40m.

We model the common errors that students make with a set of "buggy" productions. From our data, we compiled a list of student errors and analyzed what were the common errors. We found that the following list of errors was able to account for over 75% of the errors that students made. We illustrate the errors in the context of a problem, which has a correct answer of "5g+7(30-g)".

1) Wrong operator (e.g., "5g-7(30-g)")
2) Wrong order of arguments (e.g., "5g+7(g-30)")
3) Missing parentheses (e.g., "5g+7*30-g")
4) Confusing quantities (e.g., "7g+5(30-g)")
5) Missing a component (e.g., "5g+7g" or "g+7(30-g)" or "5g+30-g")
6) Omission: correct for a subgoal. (e.g., "7(30-g)" or "5g")
7) Any combinations of errors (e.g., "5/g+7*g-30" has three errors;1) the wrong order for

"g-30", 2) is missing parentheses around the 30-g, and 3) the "5/g" uses the division instead of multiplication.)

Consider what a good human tutor would do when confronted with a student who wrote what is listed in the 7th item above. Perhaps the tutor would realize that there are multiple errors in the student's answer and decide to tackle one of them first, and plan to deal with the other ones after finishing the first. In contrast, a traditional model-tracing tutor could fire three different bug rules that would generate three different bug messages and then display all three to the student. This seems to make the tutor appear more like a compiler spitting out error messages. ATM deals with each of the errors separately. Dealing with more than one error occurring at the same time (such as the 7th item in the list above), is something that Anderson's traditional model-tracing tutors do not do well, and that is probably due to the fact that the pedagogical response of such tutors is usually a buggy message. This is not to say that model-tracing tutors have *never* dealt with more than one student error occurring simultaneously; some cognitive modelers have tried to compensate for the architecture's lack of support for more than one error at a time, by writing single rules that will model two errors occurring at the same time. However, this makes the modeling work even harder.

Ms. Lindquist's Tutorial Model

Now we will look at the components of the tutorial model shown in Figure 2. A fundamental distinction in the intelligent tutoring system is between the student model, which does the diagnosing, and the tutorial model, which does everything else. The tutorial model is implemented with 77 production rules (Our use of a production system for tutorial modeling is similar to Freedman's (2000)). Some of these production rules are the selection rules shown in Figure 3, which do the selection of what type of response to make. Other rules do different things. For instance, some rules specify how to implement a particular tutorial strategy while others know when to splice in positive feedback.

Since using a tutorial strategy involves asking a series of questions, we will first state the questions Ms. Lindquist currently knows how to ask a student.

Tutorial Questions

Each example is illustrated in the context of the student working on the following problem: "Ann is in a rowboat in a lake. She is 800 yards from the dock. She then rows for "m" minutes back towards the dock. Ann rows at a speed of 40 yards per minute. Write an expression for Ann's distance from the dock." Ms. Lindquist currently has the following tutorial questions:

1) Q_symb: Symbolize a given quantity ("Write an expression for the distance Anne has rowed?")
2) Q_compute: Find a numerical answer ("Compute the distance Anne has rowed?")
3) Q_articulate: Write a symbolization for a given arithmetic quantity. This is the articulation step. ("How did you get the 120?")
4) Q_generalize: Uses the results of a Q_articulate question ("Good, Now write your answer of 800-40*3 using the variables given in the problem (i.e., put in 'm')")
5) Q_represents_what: Translate from algebra to English ("In English, what does 40m represent?" (e.g., "the distance rowed so far"))
6) Q_articulate_verbal: Explain in English how a quantity could be computed from other

quantities. (We have two forms: The reflective form is "Explain how you got 40*m" while the problem-solving form is "Explain how you would find the distance rowed?")

7) Q_decomp: Symbolize a one-operator answer, using a variable introduced to stand for a sub-quantity. ("Use A to represent the 40m for the distance rowed. Write an expression for the distance left towards the dock that uses A.")
8) Q_substitute: Perform an algebraic substitution ("Correct, that the distance left is given by 800-A. Now, substitute "40m" in place of A, to get a symbolization for the distance left.")

You will notice that questions 1, 3, 4, and 8 all ask for a quantity to symbolize. Their main difference lies in when those questions are used, and how the tutor responds to the student's attempt. Questions 5 and 6 ask the student to answer in English rather than algebra. To avoid natural language processing, the student is prompted to use pull down menus to complete this sentence "The distance rowed is equal to <noun phrase> <operator> <noun phrase>." The noun phrase menu contains a list of the quantity names for that problem. The operator menu contains "plus", "minus", "times" and "divided by." Below we will see how these questions can be combined into multi-step tutorial strategies.

Tutorial Agenda

The tutorial agenda is a data structure that operates somewhat like a stack. It is used to keep track of the current focus. It includes the questions that have been asked already of the student but are still awaiting a correct response, as well as questions that the tutor plans to ask but has not yet done so. The question at the top of the agenda represents the current question that the student was just asked. If the tutor invokes a tutorial strategy, it places the new question on the agenda to be asked. As students answer questions, they are removed from the agenda.

Tutorial Reasoning: Dynamic Scaffolding

A diagnosis is passed from the student model to the tutorial model. If the student's response is correct, the system pops that question off the agenda. However, if it is not, the dynamic scaffolding procedure requires that for each error the student made, the system come up with a plan to address it. Dynamic scaffolding is based upon the fact that human tutors tend to ask questions related to incorrect aspects of the student's answer. This *error localization* communicates valuable information to the student by focusing the student's attention on a single aspect of what might have been a complicated problem-solving process. The dynamic scaffolding procedure can also give positive feedback on correct aspects of the student's reasoning when appropriate. The dynamic scaffolding procedure does the error localization and then passes responsibility to the selection rules to determine what is the most pedagogically effective tutorial strategy to employ for the given situation. The next section details the options Ms. Lindquist has.

Tutorial Strategies

This section will show several different tutorial strategies that Ms. Lindquist can use. Some strategies we observed that the human tutor used seemed to apply only if the student made a particular type of error and we call such strategies *Knowledge Remediation Dialogs* (KRD). Other strategies the tutor used were more broadly applicable and we call such strategies *Knowledge Construction Dialog* (KCD). (We borrow the term *knowledge construction dialog* from

VanLehn.) Both KCD and KRD invoke multi-step plans to deal with particular errors, however the KRD is only applicable if the student has made a particular type of error. For instance, a dialog about the role of order of operations shown in Figure 5, would be a KRD, because it applies only in the case that the student's error was to forget parentheses. However, the concrete articulation strategy is a KCD, because it can be used no matter which specific error type might have occurred. Since KRDs apply in fewer situations, we have first focused on authoring KCDs, and have implemented only one of the KRDs we observed the experienced tutor use. That KRD is applicable when the student has made an *error of omission,* by which we mean that the student correctly symbolized only a piece of the problem. For example, suppose the student was supposed to say "800-40m" but instead said "40*m", the tutor would digress using the one-step KRD that asks the student to identify what the "40*m" represents, and once the student has clarified what the 40*m represents, the student is then asked to symbolize the whole quantity again. ATM has four different types of responses. Each of these is shown in Table 4, illustrated with an example from the Ms. Lindquist tutor.

Table 4

The four different types of tutorial responses: hints, buggy-messages, KRD and KCD

	Two types of Tutor Reponses	
	Tell the student something	**Ask a New Question(s)**
Always Applies	Traditional model-tracing hint: e.g., "You need to use 40 and 'm' to find the distance rowed."	Use one of the four KCDs such as the three-step *Concrete Articulation Strategy*
Applies selectively	Traditional model-tracing buggy-feedback message: e.g., "You are missing parentheses around the '30 G'."	Use a KRD such as shown in Figure 5 that only applies when the student has forgotten parentheses.

Table 5

An example of a knowledge remediation dialog. Note that the dialog is focused on the location of the error, as well as the type of error.

Student: **The answer is 5*g+7*30-g.**
Tutor: The 5*g is correct for the amount bagging groceries, but the amount delivering newspapers is not 7*30-g. Let me ask you this about the rules for order of operations. Which do you do first, multiplication or subtraction?
Student: **multiplication**
Tutor: Correct. In this problem, do you want the multiplication done first or the subtraction in the expression 7*30-g?
Student: **The subtraction.**
Tutor: Good- so what do you need to use to indicate that the subtraction should be done first?
Student: **Parentheses**.
Tutor: Good. So try this question again, but this time use parentheses. …

We have been using the term *selection rule* to describe the rules that determine the best tutorial responses, given the entire context so far. Because this field is so new, and tutoring is so complicated, our selection rules are currently simple heuristics, which will need to be refined by

further research. For instance, when the system has multiple different responses to choose between, its selection rules will try to put them in the following order; KRD, Buggy-Message, KCD, and finally hint. The heuristic for ordering them in this manner is to respond with the response that takes into account as much context as possible (KRD & Buggy Message). The second heuristic is to use a tutorial strategy (KRD or KCD) before using a buggy message or hint, because we would rather ask a question than give a hint. These heuristics are examples of selection rules.

We will now look at four different tutorial strategies (all KCDs) that Ms. Lindquist uses; 1) "Concrete Articulation", 2) "Introduced Variable", 3) "Explain in English First" and finally 4) "Convert the Problem into an Example to Explain."

Concrete Articulation Strategy

Our experienced human tutor used the concrete articulation strategy on 44% of the problems she tutored. This strategy uses three questions of type Q_compute, Q_articulate and Q_generalize. The right-hand side of Table 3 shows an example dialog generated by Ms. Lindquist, which closely parallels the experienced human tutor dialog. We will walk through what Ms. Lindquist did here and illustrate both dynamic scaffolding as well as the particular tutorial strategy that Ms. Lindquist used. First, the student model diagnosed the student's response ("s/m+b") as being partially correct (i.e., the student understood that the total trip time was the time of the break, "b", plus the time actually on the bikes) and partially wrong (i.e., the system noted the bug of reversing the order of the arguments (i.e., "m/s" vs. "s/m"). As part of the dynamic scaffolding routine, the system needed to come up with a plan to address the error related to the time on bikes. (This is similar to the way the human tutor also ignored the part of the problem related to the time for the break and instead focused on the goal of getting the actual time on bikes.) In this example, the tutorial model chose to use the *concrete articulation* strategy. Therefore, the tutorial model generated questions for the *compute*, *explain/articulation* and *generalization* steps and placed these three questions on the agenda.

After the student answers the compute question correctly (at S2), the tutor decides to continue with its plan. However, if the student had typed 100/20 instead of 5, then both the compute and explain questions would be removed from the agenda and the tutor would skip to the generalize step. An example of this sort of flexibility to interpret the student's response broadly is demonstrated by the tutor's response to "b+m/s" (at S4). The question at S4 was actually asking for the student to answer "m/s". Only a pedantic tutor would not accept that answer as it indicates an understanding that subsumes the understanding required for the posed question.

To show an example of how the previous situation could have come out differently, let's look at the following dialog.

T1: Hello. [Doing the "bike-trip" problem] Please write an expression for the total time the trip took. [Q_symb]

S1: **help!** [Student hit the help button]

T2: Let me ask you an easier question. Please compute the time actually on bikes if instead the speed was 20 miles per hour, the time of the break was 2 hours, and the distance ridden was 100 miles? [Q_compute]

S2: 7

T3: Correct. What math did you do to get that 7? [Q_articulate]

S3: 2+5

T4: Okay, but how did you get the 5? [Q_articulate again but this time for the 5.]
S4: **100/20**
T5: Correct. Now, let's try this question again. What math did you do to get the 7? [Q_articulate]
S5: **2+100/20**
T6: Correct. Now write your answer of '2+100/20' using the variables given in the problem (i.e., put in 'm', 'b' and 's')? [Q_generalize]
S6: **b+m/s**
T7: Correct.

This example is interesting in the way Ms. Lindquist responds at T4 with what we call an embedded sub-dialog. In this case, the student model reported that the student's answer of "2+5" is a partially complete articulation but that the student didn't spell out where the "5" came from. The tutorial model is able to "dig-down" into the student's reasoning with an embedded sub-dialog asking for an explanation of where the 5 came from. Once that was accomplished, the tutor could proceed with the rest of the plan (As indicated by saying "Now, let's try this question again.")

Croteau, Heffernan & Koedinger (2004) and Freybeger, Heffernan & Ruiz (2004) looked at tutorial log files from hundreds of students who were tutored by Ms. Lindquist and tried to find the best fitting cognitive models to explain students' responses. Croteau et al (2004) was interested in the question of whether there is evidence that an *articulate* skill is what makes writing expressions in the language of algebra difficult. This study found statistically significant evidence that students were finding "articulating" more difficult than "comprehending" word problems.

Introduced Variable Strategy

A second example of a tutorial strategy is what we call the *Introduced Variable Strategy*, which we observed our human tutor employ on close to half of the problems she tutored. This strategy uses both Q_decomp and Q_substitute questions as shown below.

T1: Hello. Adam, Stan and Doug spent the day picking strawberries. Adam picked twice as much as Stan and Doug picked 7 more kilograms than Adam. Suppose Stan picked "s" kilograms of strawberries. Stan and Doug cooperated to make a pie. Please write an expression for the combined number of kilograms for Stan and Doug.
S1: 7+(2*s) [This answer is partially correct but should be s+7+(2*s)]
T2: No. But, "7+(2*s)" is correct for the number of kilograms Doug picked. We will come back to the combined number of kilograms for Stan and Doug. Let me try to break this down for you. If "C" were the number of kilograms Doug picked then what would be the combined number of kilograms for Stan and Doug? [This is the Q_decomp question]
S2: C+s
T3: Correct. Now, let's try this question again: You are correct that "s+C" is the combined number of kilograms for Stan and Doug. Now, can you state this in terms of the numbers given in the problem? [This is the Q_substitute question]
S3: 7+(2*s)+s
T4: Correct.

Our previous research (Heffernan & Koedinger, 1998) uncovered evidence that suggested that this strategy could be effective. Because students have difficulty writing expressions that have

multiple variables, by introducing the new variable (i.e., C) at T2, the tutor reduces the number of operators the student has to write. Only after the student has gotten confirmation (that you need to add the amount Doug picked and Stan picked, using "C+s") do we ask the student to put it all together by substituting in the correct expression for "C" (i.e., T3).

"Explain in English" Strategy

Mathematical communication is increasingly seen as a goal for students, and in particular translating between an algebraic representation, an English verbal representation, and a concrete representation of numerical instances (Schoenfeld et al., 1992, Nathan & Koedinger, 2000). Other researchers are also interested in an ITS that can elicit explanations from students (e.g., Conati et al., 1997.) Our experienced human tutor would often ask the student to explain how to compute a quantity in English and only afterward, ask the student to say it in symbols (Heffernan, 2001). Therefore, we have implemented a strategy that is based on the hypothesis that students will learn more if they are first asked to explain in English how to compute a quantity, before trying to symbolize it. The following example shows this strategy:

T1: Hello. [Doing the "bike-trip" problem] Please write an expression for the total time the trip took. [Q_symb]
S1: **m/s**
T2: What does "m/s" represent (e.g., the speed on the bike?)[Q_represents_what]
S2: **The total time of the trip.**
T3: No, "m/s" represents the actual time on the bikes. Can you explain how you would find the total time of the trip using the time on the bikes? [Q_articulate_verbal]
S3: **The total time of the trip is equal to [1st menu] "the amount of time for the break" [2nd menu] "plus" [3rd menu] "the actual time on the bikes". [Composed using three pull down menus.]**
T4: Good, now say what the total trip time is in symbols [Q_symb]
S4: **b+m/s**
T5: Correct

"Convert the Problem into an Example to Explain" Strategy

This next strategy was not based on either our own empirical work, nor was it based on what the experienced tutor did. Instead, this strategy was inspired by literature and the suggestions that many other researchers have made about what makes for effective tutoring. For instance, Zhu & Simon (1987) suggest a similar strategy of presenting students with worked out examples. See also Renkl (1997), VanLehn (1998), Sweller & Cooper (1985) for related studies of student use of worked examples. Catrambone (1996) also suggests that learning from examples can be improved by asking questions that focus on the goal structures of the domain. Also, to the degree that this strategy focuses on asking students to explain the quantitative structure in English (using pull-down memus), rather then simply focusing on problem solving, the literature suggests that the *self-explanation effect* (Chi, 1996) should make this an effective strategy. For instance, Chi (1996), Conati, Larkin & VenLehn (1997), Mwangi & Sweller's (1998), and Aleven, Koedinger, & Cross (1999) have all proposed, and in some cases demonstrated that, asking students to explain can be beneficial. Most concretely of all, this strategy also gets at the goal of practicing

translation from the algebraic representation to the verbal representation, which Corbett McLaughlin, Scarpinatto, & Hadley (2000) showed to be effective for learning symbolization. Here is an example dialog:

T1: [Doing the "bike-trip" problem] Please write an expression for the total time the trip took. [Q_symb]
S1: **m*s-b**
T2: Hmm. No. The answer is actually "b+m/s". Let me ask you some questions to help you understand it. Can you explain where the "m/s" came from [Q_articulate_verbal]
S2: **The "m/s" is equal to the distance ridden divided by the speed ridden.**
T3: Correct. What does "m/s" represent? [Q_represents_what]
S3: **The time actually on bikes**
T4: Correct. Can you explain where the "b+m/s" came from? [Q_articulate_verbal variant]
S4: **The "b+m/s" is equal to the hours for break plus the time actually on bikes.**
T5: Correct. What does "h+m/s" represent? [Q_represents_what]
S5: **The total time the trip took**
T6: Correct.

Notice how at T2, the answer to the original question is given, and then the student is asked to explain the answer by translating the components back into English.

This completes the review of the strategies implemented in Ms. Lindquist.

EMPIRICAL RESULTS

Although this is a descriptive paper about the Ms. Lindquist architecture, we wanted to mention the results of a few of the evaluations that were done with Ms. Lindquist. These evaluations can be studied in depth in Heffernan (2003), Heffernan & Croteau (2004) and Mendicino, Heffernan & Razzaq (submitted).

Comparison of the "Concrete Articulation" strategy to "Cut to the Chase"

We focused this analysis on students who used Ms. Lindquist as part of a class assignment. We analyzed the classes of one teacher who sent about 76 middle school students (Heffernan & Croteau, 2004). The experimental condition received the "Concrete Articulation" strategy and the control condition was simply told the answer if they answered incorrectly and moved on to the next problem. The interaction between condition and learning gain was statistically significant with an effect size of 0.56 standard deviations. This supports the hypothesis that students do learn more in the experimental condition, even though they did significantly fewer problems.

Ms Lindquist vs. classroom instruction

In a study done by Mendicino, Heffernan & Razzaq (submitted), Ms. Lindquist was compared to both: 1) classroom instruction and 2) Computer Aided Instruction (CAI). This work tried to quantify the "value-added" of CAI over classroom instruction, versus the "value-added" of ITS (in the form of Ms. Lindquist) on top of CAI.

Both computer-based versions outperformed the classroom teachers, replicating Kulik (1994) studies showing benefits for computer instruction compared to traditional classroom

controls. The ITS did outperform CAI (measured in terms of effect size was about .4 standard derivations) suggesting that the more intelligent version was more effective at promoting learning. This experiment also replicated the motivational results reported in Heffernan (2003) where students getting the more intelligent version would persist longer.

Motivational benefits for using Ms. Lindquist

We analyzed 623 student files (see Heffernan, 2003) in an experiment with three different experimental conditions represented by the tutorial strategies mentioned earlier and a control condition which told students the answer when they got it wrong and proceeded to the next problem. Of the 623 students analyzed, 47% of the 225 that received the control condition dropped out, while only 28% of the other 398 dropped out. This difference was statistically significant. There was no statistically significant difference between the drop-out rates of the three experimental conditions. We conclude that, as far as from a motivational point of view, the intelligent feedback was superior at getting students to persist in tutoring.

DISCUSSION

It is interesting to note that in the last few years there has been an increase in interest in building dialog-based systems. However, dialog systems are not new; Carbonell (1970) built one of the early dialog-based computer tutors over 30 years ago. Since that time, many educational technologies have instead relied on elaborate graphical user interfaces (GUI) that reify parts of the problem solving process (e.g., the reification of subgoals by Corbett & Anderson, 1995). One possible benefit of dialog-based systems is that students do not have to spend time learning a new interface. This seems particularly important if the tutoring system has multiple different tutorial strategies that encourage different ways of solving problems. Therefore, the student does not have to learn multiple different GUIs for each different method.

We have released Ms. Lindquist onto the web at www.AlgebraTutor.org, where it has been used by thousands of students and teachers. Ms. Lindquist has also won various industry awards from teacher related web sites (e.g., the National Council of Teachers of Mathematics). So far, we have learned that the dialogs that Ms. Lindquist has with students can lead to better learning, compared to simply telling students the answer as well as the fact that students appear to get motivated (Heffernan & Croteau, 2004). Future work will focus on examining if the benefit of this type of tutoring is worth the additional time these dialogs require.

While Anderson's model-tracing development system was designed to allow the tutor to **tell** students how to get back on track, the ATM architecture is designed to **ask** students questions, which is more like what human tutors do. However, it remains to be seen if the ATM architecture will enable the building of tutors that are more effective than model-tracing tutors. We plan to address this question by comparing the Ms. Lindquist tutoring system to a control version that uses only the traditional model-tracing forms of feedback (buggy messages and hints). We are also currently running experiments comparing the effectiveness of the different tutorial strategies Ms. Lindquist has. We are also interested in generalizing this architecture further by building a set of authoring tools for content experts to be able to author similar intelligent tutoring systems.

Later, we want to learn "Under what conditions is it best to use tutorial strategy X versus tutorial strategy Y?" For example, it might be best to use the concrete articulation strategy for problems that include only a few arithmetic operations. Alternatively, maybe there is utility in using multiple different strategies. Answers to these questions can be found by systematically

experimenting with the selection rules used by the system. Arroyo et al. (2000) provides a nice example of a selection rule; students who score low on a Piagetian test perform better if given instruction that is more concrete, while high scoring students learn better with instruction that is more formal. Arroyo et al. (2001) have also found evidence suggesting boys are less likely to read hint messages and benefit from less interactive hints. We plan to use Ms. Lindquist to discover progressively more detailed selection rules. As we run more experiments, refining our selection rules and adding new tutorial strategies, we will be creating a concrete theory of tutoring for symbolization that makes specific recommendations. Some of the tutor's behaviors will be shown to be more helpful than others. Of course, we will never reach the perfect tutoring model, but by making our theories about tutoring concrete, we accumulate a body of useable knowledge about what makes for good tutoring.

CONCLUSION

McArthur et al. (1990) criticized the model-tracing architecture “because each incorrect rule is paired with a particular tutorial action (typically a stored message)" and argued for a more strategic tutor. The ATM architecture and the Ms. Lindquist tutor address this criticism. The main difference between ATM and Traditional Model-Tracing is the incorporation of a tutorial model. Whereas traditional model-tracing tutors generate all their feedback from text templates that are inside the rules in the cognitive model, the ATM architecture generates a plan (usually involving multiple new questions to ask the student) for each error the student made. The model-tracing architecture does not have a way of encoding new general pedagogical knowledge, beyond that inherent in the architecture (such as giving feedback in response to errors). In summary, The ATM architecture allows Ms. Lindquist to combine the student modeling of traditional model-tracing tutors with a model of tutorial dialog based on an experienced human tutor including such features as positive and negative feedback, multiple tutorial strategies, with embedded sub-dialogs, as well as traditional buggy messages and hints.

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