

## COMP 280 : Assignment 6

due: Thursday, March 16, 2000

1. (2 pts) How many positive integers less than or equal to 100 are relatively prime to 100? (Two integers are relatively prime if they have no factors in common, other than 1).
2. (3 pts) How many solutions are there to the following equation, where each  $x_i$  must be a positive (non-zero) integer?

$$x_1 + \dots + x_j = k$$

Assume that  $k \geq j$ .

3. (4 pts) Find a closed form for the following recurrence relations for  $n \geq 2$ :

(a)  $a_n = a_{n-1} + 6a_{n-2}$  when  $a_0 = 3$  and  $a_1 = 6$

(b)  $a_n = 6a_{n-1} - 8a_{n-2}$  when  $a_0 = 4$  and  $a_1 = 10$

4. (2 pts) Section 5.3, problem 9b.
5. (3 pts) Given positive integer  $n$ , how many strictly increasing sequences of positive integers are there which end in  $n$  and do not contain any consecutive integers? For instance, when  $n = 1$  there is just one: the singleton sequence  $\langle 1 \rangle$ . When  $n = 4$  there are three:  $\langle 1, 4 \rangle$ ,  $\langle 2, 4 \rangle$ , and  $\langle 4 \rangle$ . Express your answer as a recurrence relation.