COMP 280: Assignment 2 - Sample Solutions

1. • 1 ∈ S₂
   • If x ∈ S₂, then 2x ∈ S₂
   • 1 ∈ S₁
   • If x ∈ S₃, then 3x ∈ S₃
   • S = S₂ ∪ S₃

2. (a) If PS(S) represents the powerset of S:
   • ∅ ∈ PS(S)
   • If x ∈ S and T ∈ PS(S), then (x) ∪ T ∈ PS(S)

   (b) Formula: |PS(S)| = 2^{|S|}
   Proof, by induction on the size of S:
   Base case: assume S is empty. Then PS(S) = {∅}. So |PS(S)| = 1 = 2^0 = 2^{|S|}.
   Inductive step: assume |PS(S)| = 2^{|S|}. If T = S ∪ {x}, and x ∉ S, let’s prove that |PS(T)| = 2^{|T|}.
   Suppose that, to create PS(T), we first use the first axiom above to add the empty set, then go
   through all the elements of S in turn using each time the second axiom, then finally consider x
   and use the second axiom one last time for it.
   Just before considering x, the powerset of T will contain the same sets as the powerset of S,
   because all and only the elements of S will have been considered so far, plus the empty set, just
   as for the powerset of S. Now we consider x. For each set U ∈ PS(T), we will create a new set
   {x} ∪ U, and add it to PS(T). So the number of sets in PS(T) will double. So:
   |PS(T)| = 2 * |PS(S)| = 2 * 2^{|S|} (by the induction hypothesis), so |PS(T)| = 2^{|T|}. But T = S ∪ {x}
   and x ∉ S, so |T| = |S| + 1, which proves |PS(T)| = 2^{|T|}, and completes the proof.

3. (a) If S is a stack, then, by definition of a stack, S is either empty stack or push(e, S') for some e and
   some stack S’.
   If S is empty stack, then, by the fifth axiom, empty stack? (S) is true. Then either empty stack? (S)
   or nonempty stack? (S) is true (since empty stack? (S) is true, which is enough to prove the whole
   statement true, regardless of what nonempty stack? (S) might be).
   If S is push(e, S’), then, by the sixth axiom, nonempty stack? (S) is true. Which is again enough
   to prove the statement to be true, using the existing axioms.

   (b) No axiom directly tells the value of nonempty stack(new stack()). By definition of new stack, new-
   stack() is equal to empty stack. So the proposed axiom is equivalent to nonempty stack(empty stack)
   is false. But no axiom tells the value of nonempty stack(empty stack), and no operation can be
   further applied. So the proposed axiom can not be proved using the existing axioms.

4. For some database D, year Y, and number N:
   • rank(Y first class(D) D) = 1
   • rank(Y second class(D) D) = 2
   • rank(Y third class(D) D) = 3
   • rank(Y record contrib(Y N ∅)) = 1
   • If rank(Y D) < rank(Y Y D), then amount contrib(Y Y D) > amount contrib(Y Y D), else
     amount contrib(Y Y D) < amount contrib(Y Y D).
   • If amount contrib(Y Y D) < amount contrib(Y Y D), then rank(Y Y D) > rank(Y Y D), else
     rank(Y Y D) < rank(Y Y D).
   • amount contrib(Y record contrib(Y N ∅)) = N
   • record contrib(Y amount contrib(Y N ∅) D) = D
   • record contrib(Y Y N1 record contrib(Y Y N2 D)) = record contrib(Y Y N2 record contrib(Y Y N1 D)).