1. (4 pts) A simple airline database consists of a relation direct_flight(from_city, to_city). For each of the following queries over this database, either express it as a conjunctive query (no negation) or prove that it cannot be written as a conjunctive query.

   (a) There exists a route with only two stopovers from Houston to Paris.
   (b) All routes from Houston to Paris contain two stopovers.

2. (3 pts) An employee database contains a relation EmplInfo with attributes Name, Division, Manager, Office, and Extension. The database has a functional dependency from Division → Manager. If the database also has a functional dependency from Name → Manager, must it also satisfy a functional dependency from Name → Division? Provide a proof or a counterexample.

3. (14 pts) In class, we considered the film database query “pairs of actors and directors such that the actor appeared in a film directed by the director”. One proposal was the conjunctive query

   \[\text{ans}(\text{Dir}, \text{Actor}) \leftarrow \text{Movies}(\text{Title}, \text{Dir}, \text{Actor}),\]

   which we determined was incorrect because a single film might have multiple directors. Instead, we decided that the correct query is

   \[\text{ans}(\text{Dir}, \text{Actor}) \leftarrow \text{Movies}(\text{Title}, \text{Dir}, \text{Actor}), \text{Movies}(\text{Title}, \text{Dir'}, \text{Actor})\]

   (a) (3 pts) Consider a movie database with a functional dependency from Title → Director. Are the two above queries equivalent in this case? Prove your answer or provide a counterexample.

   (b) (3 pts) Consider a movie database with a join dependency

   \[\text{Movies} \bowtie\{\{\text{Title, Actor}\}, \{\text{Title, Director}\}\}\]

   Are the two above queries equivalent in this case? Prove your answer or provide a counterexample.

   (c) (3 pts) Consider a movie database with a join dependency

   \[\text{Movies} \bowtie\{\{\text{Director, Title}\}, \{\text{Director, Actor}\}\}\]

   Are the two above queries equivalent in this case? Prove your answer or provide a counterexample.

   (d) (5 pts) Consider a film database with a functional dependency Title → Director and a join dependency

   \[\text{Movies} \bowtie\{\{\text{Title, Actor}\}, \{\text{Title, Director}\}\}\]

   Is either dependency redundant in the presence of the other? Specifically, does the functional dependency imply the join dependency or vice-versa? For each case, provide a proof or counterexample to justify your answer.