1. (4 pts) Prove or provide a counterexample for each of the following statements about sets:
   (a) (2 pts) \((A \cap B) \cup C = A \cap (B \cup C)\) if and only if \(C \subseteq A\).
   (b) (2 pts) \(A \cap (B \cup C) = (A \cup B) \cap (A \cup C)\)

2. (6 pts) We have discussed the following two implementations of multisets:

   A multiset is either
   - empty, or
   - \((\text{cons} \ E \ MS)\), where \(E\) is an element and \(MS\) is a multiset

   An element Rec is a structure \((\text{make-elt elt count})\) where \(elt\) is an element and \(count\) is a number.

   A multiset is either
   - empty, or
   - \((\text{cons} \ ER \ MS)\), where \(ER\) is an element Rec and \(MS\) is a multiset that does not have an
     element Rec for the element in \(ER\).

   For each representation:
   (a) Write a Scheme program to compute the union of two multisets.
   \[
   \text{: union} \ : \ \text{multiset} \ \text{multiset} \ \rightarrow \ \text{multiset}
   \]
   (b) (3 pts) Provide and prove a statement that your union program is correct.

3. (10 pts) Rice University maintains information about the assignments of students to its eight colleges. Each college can accommodate a large but limited number of students. The university performs several queries on this information:
   - How many students are assigned to a given college?
   - Which students are assigned to a given college?
   - How many spaces are available in a given college?

   (a) (4 pts) Provide two different data models for the housing information. Briefly discuss the advantages and disadvantages of each model. Warnings:
      - Your two models should not be simple variations of one another.
      - Describe models, not implementations.

   (b) (6 pts) Assume you are using the following data definitions for the housing information:

   A RiceCollege is one of "Baker", "Brown", "Hansen", "Jones", "Lovett", "SidRich", "Weiss", "WillRice"

   A StudentRec is a structure \((\text{make-student name college})\)
   - name is a symbol and college is a RiceCollege.

   A HousingInfo is either
   - empty, or
   - \((\text{cons} \ SR \ HI)\) where \(SR\) is a StudentRec and \(HI\) is a HousingInfo.

   The program for computing the students in a given college has the following contract:

   \[
   \text{: InCollege} : \ \text{HousingInfo} \ \text{RiceCollege} \ \rightarrow \ \text{listof StudentRecs}
   \]
   - returns a list of records of all students assigned to the named college

   Provide two Scheme implementations of InCollege, one using structural recursion and one using an accumulator. Prove that the two programs produce the same lists of student records; the statement that you prove should be as precise as possible. Clearly state and prove all intermediate results required for your proof.