#1 True or False

a) Regular languages are recursive                      TRUE    FALSE
b) Context free languages are recursively enumerable (r.e.) TRUE    FALSE
c) Recursive languages are r.e                        TRUE    FALSE
d) R.e. languages are recursive                        TRUE    FALSE

#2. a) Show computations with 000111 and 101 on the following Turing Machine

<table>
<thead>
<tr>
<th>State</th>
<th>0</th>
<th>1</th>
<th>X</th>
<th>Y</th>
<th>□</th>
</tr>
</thead>
<tbody>
<tr>
<td>q0</td>
<td>(q0, R)</td>
<td>(q1, X, R)</td>
<td>-</td>
<td>-</td>
<td>(q3, Y, R)</td>
</tr>
<tr>
<td>q1</td>
<td>(q1, 0, R)</td>
<td>(q2, Y, L)</td>
<td>-</td>
<td>(q1, Y, R)</td>
<td>-</td>
</tr>
<tr>
<td>q2</td>
<td>(q2, 0, L)</td>
<td>-</td>
<td>(q0, X, R)</td>
<td>(q2, Y, L)</td>
<td>-</td>
</tr>
<tr>
<td>q3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(q3, Y, R)</td>
<td>(q4, □, R)</td>
</tr>
<tr>
<td>q4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

000111
q0 |-000111 □ ... q0 |-101
- q000111 □ ... q0101
-Xq100111
-X0q10111
-X0q1111
-X0q20Y11
-Xq200Y11
-q2X00Y11
-Xq00Y11
-Xq20Y11
-XXq10Y11
-XX0q1Y11
-XX0Yq111
-XX0q2YY1
-XXq20YY1
-Xq2X0YY1
-XXq00YY1
-XXq2YY1
-XXq0YY1
-XXQ1YY1
-XXYq1Y1
-XXYYq111
-XXYYq2YY
-XXq2YY
-XXq2XYYY
-XXq0YYY
-XXYYq3YY
XXXYYq₃Y
XXXYYYq₃
XXXYYYBq₄
halt
q₄ is a final state

b) What is L(M) (you’ll have to guess)

“Looks like” \{0ⁿ₁ⁿ | n > 0\}

#3. Construct a Turing Machine to compute \{w w^R | w \in \{0,1\}^*\}

a) Show pseudo-code that describes how the TM operates

If symbol is 0
  write X
  enter a "branch" that iterates to the 1ˢᵗ Blank,X,Y
if last symbol is 0
  go back to the beginning, repeat
(Similar for 1)
(Accept if read X or Y at 1ˢᵗ step)

b) Create the actual transitions

<table>
<thead>
<tr>
<th>State</th>
<th>_</th>
<th>0</th>
<th>1</th>
<th>X</th>
<th>Y</th>
<th>_</th>
</tr>
</thead>
<tbody>
<tr>
<td>\rightarrow q₀</td>
<td>(q₀, _ , R)</td>
<td>(q₁,X,R)</td>
<td>(q₄,Y,R)</td>
<td>(q₇,X,L)</td>
<td>(q₇,Y,L)</td>
<td>_</td>
</tr>
<tr>
<td>q₁</td>
<td>(q₁,0,R)</td>
<td>(q₁,1,R)</td>
<td>(q₂,X,L)</td>
<td>(q₂,Y,L)</td>
<td>(q₂, _ , L)</td>
<td></td>
</tr>
<tr>
<td>q₂</td>
<td>(q₃,X,L)</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td></td>
</tr>
<tr>
<td>q₃</td>
<td>(q₃,0,L)</td>
<td>(q₃,1,L)</td>
<td>(q₀,X,R)</td>
<td>_</td>
<td>_</td>
<td></td>
</tr>
<tr>
<td>q₄</td>
<td>(q₄,0,R)</td>
<td>(q₄,1,R)</td>
<td>(q₅,X,L)</td>
<td>(q₅,Y,L)</td>
<td>(q₅, _ , L)</td>
<td></td>
</tr>
<tr>
<td>q₅</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>(q₆,Y,L)</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td>q₆</td>
<td>(q₆,0,L)</td>
<td>(q₆,1,L)</td>
<td>_</td>
<td>(q₀,Y,R)</td>
<td>_</td>
<td></td>
</tr>
<tr>
<td>q₇</td>
<td>(q₈, _ , R)</td>
<td>_</td>
<td>_</td>
<td>(q₇,X,L)</td>
<td>(q₇,Y,L)</td>
<td>_</td>
</tr>
<tr>
<td>*q₈</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
</tr>
</tbody>
</table>

c) Show your TM processing (i) 1001, (ii) 101 and (iii) 110

q₀ \rightarrow 1001 \rightarrow q₀ 001 \rightarrow Yq₄001 \rightarrow Y₀q₄01 \rightarrow Y₀q₄1 \rightarrow Y₀q₄1
q₀ \rightarrow Y₀q₁q₄ \rightarrow Y₀q₆0q₁ \rightarrow Yq₇₆q₄ \rightarrow Yq₆₀q₄ \rightarrow Yq₆₀q₄ \rightarrow q₆q₆0Y \rightarrow q₆Y₀0Y
q₀ \rightarrow Yq₆q₄ \rightarrow Y₈q₄0Y \rightarrow Y₈q₄1Y \rightarrow Y₈q₄₂Y \rightarrow Y₈q₄₂Y \rightarrow Y₈q₄₂Y \rightarrow Y₈q₄₂Y \rightarrow Y₈q₄₂Y \rightarrow Y₈q₄₂Y
q₀ \rightarrow Yq₄XY \rightarrow Yq₄XY \rightarrow Yq₄XY \rightarrow Yq₄XY \rightarrow q₇YXXX \rightarrow Yq₄XY \rightarrow Yq₄XY

q₀ \rightarrow 101 \rightarrow q₀ 101 \rightarrow Yq₄01 \rightarrow Y₀q₄1 \rightarrow Y₀q₄1 \rightarrow Y₀q₄1
#4. Show that r.e. languages are closed under union and intersection.

**union**
Assume $L_1$ and $L_2$ are recursively enumerable. We can consider two single tape machines $M_1$ and $M_2$, which accept $L_1$ and $L_2$ respectively. We define $M$ as a single tape TM with three tracks. Track 1 will hold the input. $M$ will simulate $M_1$ using track 2. If $M_1$ halts in an accepting configuration then $M$ accepts; otherwise $M$ moves the tape head back to the left end and starts simulating $M_2$ or track 3. If $M_2$ accepts the input string then the string is accepted by $M$.

**intersection**
Assume $L_1$ and $L_2$ are recursively enumerable. We can consider two single tape machines $M_1$ and $M_2$, which accept $L_1$ and $L_2$ respectively. We define $M$ as a single tape TM with three tracks. Track 1 will hold the input. $M$ will simulate $M_1$ using track 2. If $M_1$ halts in an accepting configuration $M$ moves the tape head back to the left end and starts simulating $M_2$ or track 3. If $M_2$ also accepts the input string then the string is accepted by $M$. 