

## Homework #6

- #1. a) If  $011$  is accepted by an NPDA, it is accepted by a DPDA True False  
 b) If  $011$  is accepted by an NFA, it is accepted by a DFA True False  
 c) NPDA's can accept more languages than DPDA's True False  
 d) If a PDA accepts by final state, then it accepts by empty stack True False  
 e) If  $L$  is accepted by a dfa  $M$ , then it is accepted by a PDA,  $N$  True False

#2. Given the PDA  $P = (\{q, p\}, \{0, 1\}, \{\perp, X\}, \delta, q, \perp, \{p\})$   
 with the following transition functions:

1.  $\delta(q, 0, \perp) = \{(q, X\perp)\}$
2.  $\delta(q, 0, X) = \{(q, XX)\}$
3.  $\delta(q, 1, X) = \{(q, X)\}$
4.  $\delta(q, \varepsilon, X) = \{(p, \varepsilon)\}$
5.  $\delta(p, \varepsilon, X) = \{(p, \varepsilon)\}$
6.  $\delta(p, 1, X) = \{(p, XX)\}$
7.  $\delta(p, 1, \perp) = \{(p, \varepsilon)\}$

a) Show all reachable configurations when

a)  $w = 01$

$$(q, 01, \perp) \xrightarrow{1} (q, 1, X\perp) \xrightarrow{3} (q, \varepsilon, X\perp) \xrightarrow{4} (p, \varepsilon, \perp) \\ \xrightarrow{1} (q, 1, X\perp) \xrightarrow{4} (p, 1, \perp) \xrightarrow{7} (p, \varepsilon, \varepsilon)$$

b)  $w = 0011$

$$(q, 0011, \perp) \xrightarrow{1} (q, 011, X\perp) \xrightarrow{2} (q, 11, XX\perp) \xrightarrow{3} (p, 1, X\perp) \xrightarrow{4} (p, 1, \perp) \\ \xrightarrow{7} (p, \varepsilon, \varepsilon) \\ \xrightarrow{1} (q, 011, X\perp) \xrightarrow{2} (q, 11, XX\perp) \xrightarrow{4} (p, 11, X\perp) \xrightarrow{6} (p, 1, XX\perp) \\ \xrightarrow{6} (p, \varepsilon, XX\perp) \\ \xrightarrow{1} (q, 011, X\perp) \xrightarrow{2} (q, 11, XX\perp) \xrightarrow{3} (q, 1, X\perp) \xrightarrow{3} (q, \varepsilon, \perp) \\ \xrightarrow{1} (q, 011, X\perp) \xrightarrow{4} (p, 011, X\perp) \\ \xrightarrow{1} (q, 011, X\perp) \xrightarrow{2} (q, 11, XX\perp) \xrightarrow{4} (p, 11, X\perp) \xrightarrow{6} (p, 1, XX\perp) \\ \xrightarrow{5} (p, 1, X\perp) \xrightarrow{5} (p, 1, \perp) \xrightarrow{7} (p, \varepsilon, \varepsilon) \\ \xrightarrow{1} (q, 011, X\perp) \xrightarrow{2} (q, 11, XX\perp) \xrightarrow{6} (p, 1, XXX\perp) \xrightarrow{6} (p, \varepsilon, XXXX\perp)$$

a)  $w = 010$

$$(q, 010, \perp) \xrightarrow{1} (q, 10, X\perp) \xrightarrow{3} (q, 0, X\perp) \xrightarrow{2} (q, \varepsilon, XX\perp) \\ \xrightarrow{1} (q, 10, X\perp) \xrightarrow{4} (p, 10, \perp) \xrightarrow{7} (p, 0, \varepsilon) \\ \xrightarrow{1} (q, 10, X\perp) \xrightarrow{3} (q, 0, X\perp) \xrightarrow{4} (p, 0, \perp)$$

b) Also describe  $L(M)$

$$L(M) = \{w \text{ in } \{0,1\}^* \mid \dots\}$$

#3. Design a PDA to accept the set of all strings of 0's and 1's with an equal number of 0's and 1's.

1.  $\delta(q, 0, \perp) = \{(q, 0\perp)\}$  recording 0's
2.  $\delta(q, 1, \perp) = \{(q, 1\perp)\}$  recording 1's
3.  $\delta(q, 0, 0) = \{(q, 00)\}$  recording 0's
4.  $\delta(q, 0, 1) = \{(q, \epsilon)\}$  matching
5.  $\delta(q, 1, 1) = \{(q, 11)\}$  recording 1's
6.  $\delta(q, 1, 0) = \{(q, \epsilon)\}$  matching
7.  $\delta(q, \epsilon, \perp) = \{(p, \epsilon)\}$  accepting by empty stack (or final state!)

The PDA  $P = (\{q\}, \{0, 1\}, \{0, 1, \perp\}, T, q, \perp, \{p\})$ , where  $T$  consists of the transitions 1-7 defined above.

#4 a) Convert the grammar,  $S \rightarrow 0 S 0 \mid 1 S 1 \mid \epsilon$  to an equivalent NPDA. Show your NPDA accepting 0 1 1 0 and rejecting 0 1 1.

**First convert to GNF. Easy because grammar is small:**

$$\begin{aligned} S &\rightarrow 0 S Z \mid 1 S O \mid \epsilon \\ Z &\rightarrow 0 \\ O &\rightarrow 1 \end{aligned}$$

$$M = (\{q\}, \{0,1\}, \{S, Z, O\}, \delta, S, \Phi)$$

$\delta$

$$\begin{aligned} S \rightarrow 0 S Z &\rightarrow \delta(q, 0, S) = (q, SZ) \\ S \rightarrow \epsilon &\rightarrow \delta(q, \epsilon, S) = (q, \epsilon) \\ S \rightarrow 1 S O &\rightarrow \delta(q, 1, S) = (q, SO) \\ Z \rightarrow 0 &\rightarrow \delta(q, 0, Z) = (q, \epsilon) \\ O \rightarrow 1 &\rightarrow \delta(q, 1, O) = (q, \epsilon) \end{aligned}$$

0110

$$\begin{aligned} (q, 0110, S) &\rightarrow (q, 110, SZ) \\ &\rightarrow (q, 10, SOZ) \\ &\rightarrow (q, 10, OZ) \\ &\rightarrow (q, 0, Z) \\ &\rightarrow (q, \epsilon, \epsilon) \end{aligned}$$

## 011

Lots of dead ends here

$(q,011,S) \rightarrow (q,11,SZ)$  or  $(q,011, \epsilon)$  (dead end)  
 $\rightarrow (q,1, SOZ)$  or  $(q,11,Z)$  (dead end)  
 $\rightarrow (q, SOOZ)$  or  $(q,1,OZ) \rightarrow (q, \epsilon,Z)$  (dead end)  
 $\rightarrow (q, \epsilon ,OOZ)$  (dead end)

#5. Convert your NPDA from part 1 back to a CFG. Show your grammar generating 0 1 1 0 and not generating 0 1 1.

$\delta (q,0,S) = (q,SZ) \rightarrow S \rightarrow 0 S Z$   
 $\delta (q,1,S) = (q,SO) \rightarrow S \rightarrow 1 S O$   
 $\delta (q,0,Z) = (q, \epsilon) \rightarrow Z \rightarrow 0$   
 $\delta (q,1,O) = (q, \epsilon) \rightarrow O \rightarrow 1$