Name___________________

CS503 Homework #5

Solutions

#1. Show that the following languages are or are not context-free

a) $\{w\, w^R\, w \mid w \in \{a,b\}^* \}$

Not c-f

If L were c-f, then there is a constant, k, such that if $z \in L$ and $|z| \geq k$, the PL conditions are true.
Pick $z = a^n b^n b^n a^n a^n b^n$. Then $z \in L$ and $|z| \geq k$
So
$z = u\, v\, w\, x\, y$ with

$|v\, w\, x| \leq k$
$|v| + |y| > 0$ (i.e., not both v and x are λ)

There are two cases:

Case 1 *v w x* is completely within one of the a^n or b^n's, say the first b^n

Assume $|v| > 0$ so $v = b^p$ with $p > 0$. (Results will be similar if $|x| > 0$)

Then *u v v w x x y* = $a^n b^{n+p} b^n a^n a^n b^n$ and there is no way to split this up to be of the form: $w\, w^R\, w$

Case 2 z overlaps a's and b's or b's and a's: then either first and last w will not be the same or again no way to split this up to be of the form: $w\, w^R\, w$

b) $\{a^i b^{2i} c^j \mid i,j \geq 0\}$
c-f:

S → A C
A → a A b b | λ
C → c C | λ

c) $\{a^n b^n a^n \mid n \geq 0\}$

Not c-f
Proof similar to $\{a^n b^n c^n \mid n \geq 0\}$

d) $\{x \in \{0,1\}^* \mid \#_0(x) = \#_1(x)\}$

S → 0 S 1 | 1 S 0 | S S

#2. For each of the following languages, show it is either a) regular, b) context-free, but not regular, c) not context-free

a) $\{a^n b^m \mid n = 2m\}$

This is $\{a^{2m} b^m\}$ which is not regular by the PL
(done in class)
C-F:
S → aa S b | λ

b) $\{a^n b^{2m} \mid n, m \geq 0\}$

This is $a^{*(}bb)^*$ which is a regular expression, so regular

c) $\{a^n b^m \mid n \neq m\}$

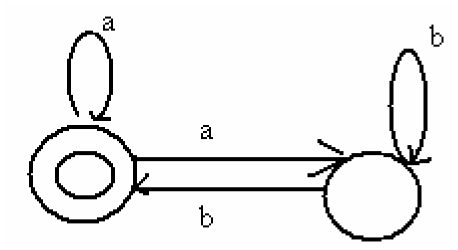
Not regular
If it were regular, then its complement would also be regular, but we know its complement which is $\{a^n b^n\}$ is not regular

c-f:
S → a S B | B
B → b B | b
generates
$\{a^n b^m \mid n < m\}$

S → A S b | A
A → a A | a
generates
$\{a^n b^m \mid n > m\}$

The union of the languages generated by these grammars is $\{a^n b^m \mid n \neq m\}$, hence it is c-f.

#3. a) Use the subset construction to convert the following nfa to a dfa



	a	b
{0}	{01}	∅
{01}	{01}	{01}
∅	∅	∅

b) Give a regular expression for L(M)

a (a ∪ b)*

#4. Prove: CFL's are closed under union, concatenation and Kleene *

<u>union</u>
Given
S_1 → … for L_1
and
S_2 → … for L_2

Create G: S → S_1 | S_2

<u>concatenation</u>
Given
S_1 → … for L_1
and
S_2 → … for L_2

Create G: S → S_1 S_2

<u>*</u>
Given
S_1 → … for L_1

Create G: S → S_1 S

#5. Prove: CFL's are not closed under intersection or complement

We know that $\{a^n b^n c^n \mid n \geq 0\}$ is not c-f, but it is the intersection of the 2 c-f languages $\{a^n b^n c^m \mid m,n \geq 0\}$ and $\{a^m b^n c^n \mid m,n \geq 0\}$, so cfl's not closed under intersection.

If cfl's closed under complement,

then because $L_1 \cap L_2 = \sim(\sim L_1 \cup \sim L_2)$, cfl's would be closed under $\cap$.