#1. Show the following languages are not regular

a) \( L = \{a^n b^n a^n \mid n \geq 0\}\)

b) \( L = \{a^i \mid i \text{ is prime}\}\)

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Proof by contradiction using Pumping Lemma

L is infinite. Assume that L is regular.
Because L is infinite, we can pick
Pick \( w = a^m b^m a^m \) where \( m \) is the constant in the PL
Then
\[ W = xyz \text{ with } |xy| \leq m, |y| \geq 1; \]
this makes xy all “a”s
Suppose |xy| = s and |y| = k \( \geq 0\)

Then \( z = a^{m-s} b^m a^m \)

Pumping y twice gives
\( a^{m+k} b^m a^m \) which is not of the form \( a^n b^n a^n \)

This is a contradiction and thus L is not a regular language.

#2. Exercise 6.1.1 a), b) and c), Page 228. Also describe L(M)

Given the PDA \( P= (\{q,p\}, \{0,1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\}) \)

with the following transition functions:

1. \( \delta(q, 0, Z_0) = \{(q, XZ_0)\}\)
2. \( \delta(q, 0, X) = \{(q, XX)\}\)
3. \( \delta(q, 1, X) = \{(q, X)\}\)
4. \( \delta(q, \varepsilon, X) = \{(p, \varepsilon)\}\)
5. \( \delta(p, \varepsilon, X) = \{(p, \varepsilon)\}\)
6. \( \delta(p, 1, X) = \{(p, XX)\}\)
7. \( \delta(p, 1, Z_0) = \{(p, \varepsilon)\}\)

Show all reachable ID’s when
a) $w = 01$

$(q, 01, Z_0) \rightarrow (q, 1, XZ_0) \rightarrow (q, \varepsilon, XZ_0) \rightarrow (p, \varepsilon, Z_0)$

$(q, 1, XZ_0) \rightarrow (p, 1, Z_0) \rightarrow (p, \varepsilon, \varepsilon)$

b) $w = 0011$

$(q, 0011, Z_0) \rightarrow (q, 011, XZ_0) \rightarrow (p, 1, XZ_0) \rightarrow (p, 1, Z_0)$

$(q, 0011, XZ_0) \rightarrow (q, 11, XZ_0) \rightarrow (p, 1, Z_0) \rightarrow (p, \varepsilon, \varepsilon)$

#3. Exercise 6.2.1 c)

Design a PDA to accept the set of all strings of 0’s and 1’s with an equal number of 0’s and 1’s.

1. $\delta(q, 0, Z_0) = \{(q, 0Z_0)\}$ recording 0’s
2. $\delta(q, 1, Z_0) = \{(q, 1Z_0)\}$ recording 1’s
3. $\delta(q, 0, 0) = \{(q, 00)\}$ recording 0’s
4. $\delta(q, 0, 1) = \{(q, \varepsilon)\}$ matching
5. $\delta(q, 1, 1) = \{(q, 11)\}$ recording 1’s
6. $\delta(q, 1, 0) = \{(q, \varepsilon)\}$ matching
7. $\delta(q, \varepsilon, Z_0) = \{(p, \varepsilon)\}$ accepting by empty stack (or final state!)

The PDA $P = (\{q\}, \{0, 1\}, \{0, 1, Z_0\}, T, q, Z_0, \{p\})$, where $T$ consists of the transitions 1-7 defined above.

#4. a) Create a PDA which accepts the same language as that generated by:

$S \rightarrow a \ A \ B \ | \ a \ B$
$A \rightarrow a \ A \ B \ | \ a \ B$
$B \rightarrow b$

c) Show a derivation of and a computation with $aaabbb$
1. \( \delta(q, \varepsilon, S) = \{(q, aAB)\} \)
2. \( \delta(q, \varepsilon, S) = \{(q, aB)\} \)
3. \( \delta(q, \varepsilon, A) = \{(q, aAB)\} \)
4. \( \delta(q, \varepsilon, A) = \{(q, aB)\} \)
5. \( \delta(q, \varepsilon, B) = \{(q, b)\} \)
6. \( \delta(q, a, a) = \{(q, \varepsilon)\} \)
7. \( \delta(q, b, b) = \{(q, \varepsilon)\} \)

Derivation (left-most):

\[ S \Rightarrow aAB \Rightarrow aaABB \Rightarrow aaaBBB \Rightarrow aaabBB \Rightarrow aaabbB \Rightarrow aaabbb \]

Computation:

\[
(q, aaabbb, S) \xrightarrow{1} (q, aaabbb, aAB) \xrightarrow{6} (q, aabbb, AB) \xrightarrow{3} (q, aabbb, aABB) \\
\xrightarrow{6} (q, abbb, ABB) \xrightarrow{3} (q, abbb, aBBB) \xrightarrow{6} (q, bbb, BBB) \xrightarrow{5} (q, bbb, bBB) \\
\xrightarrow{7} (q, bb, BB) \xrightarrow{5} (q, bb, bB) \xrightarrow{7} (q, b, B) \xrightarrow{7} (q, , \varepsilon) \]

#5. (extra) Draw a set diagram that shows the relationship between regular languages, context-free languages, ambiguous context-free languages, unambiguous context-free languages, DPDA languages, and non-context-free languages: