#1. Prove that the following is not a regular language: The set of strings of 0’s and 1’s that are of the form $w w$

**Proof by contradiction using the Pumping Lemma**

The language is clearly infinite, so there exists $m$ (book uses a $k$) such that if I choose a string with $|\text{string}| \geq m$, the 3 properties will hold.

Pick string $= 0^m 1 0^m 1$. This has length $> m$.

So, there is an $u, v, w$ such that $\text{string} = uvw$ with $|uv| \leq m$, $|v| \geq 0$.

So $uv = 0^a$, $v = 0^k$, and $w = 0^{m-n} 1 0^m 1$

And the string $uvv w$ is supposed to also be in the language.

**But** $uvv w = 0^{n+k} 1 0^{m-n} 1 0^m 1$, and a few minutes staring at this should convince that there is no way this could be of the form $w w$ (Be sure you see why).

#2. Show that the language $L = \{a^p\ p \text{ is prime}\}$ is not a regular language

**Proof by contradiction using the Pumping Lemma**

The language is clearly infinite, so there exists $k$ such that if I choose a string with $|\text{string}| \geq k$, the 3 properties will hold.

Pick $\text{string} = 1^p$

Then we can break $\text{string}$ into $uvw$ such that $v$ is not empty and $|uv| \leq k$. 
Suppose $|v| = m$. Then $|uv| = p - m$.
If the language really is regular, the string $uv^{p-m} w$ must be in the language.

**But**, $|uv^{p-m} w| = p - m + (p-m)m$
which can be factored into $(m+1)(p-m)$.
Thus this string does not have a length which is prime, and cannot be in $L$. 
This is a contradiction.
#3. Suppose $h$ is the homomorphism from \{0,1,2\} to \{a,b\} defined by $h(0) = a$; $h(1) = ab$; $h(2) = ba$.

a) What is $h(21120)$

$h(21120) = ba\ ab\ ab\ ba\ a$

b) If $L = 01^*2$, what is $h(L)$?

$h(L) = a(ab)^*ba$

c) If $L = a(ba)^*$, what is $h^{-1}(L)$?

$02^*U\ 1*0$

#4. a) Show that the question: Does $L = \Sigma^*$? for regular language $L$ is decidable.

First the question Does $L = \Phi$ is decidable: Just have to look at the dfa for $L$ to see if there is a path from the start state to a final state.

Now look at the complement of $L$, $L'$. It is decidable whether it is empty because the complement of a regular language is regular. If $L'$ is empty, then $L = \Sigma^*$; otherwise $L \neq \Sigma^*$.

A more interesting proof is that a language is empty (hence its complement is $\Sigma^*$) if and only if the related dfa accepts a string whose length is less than $k$, the number of states (Then we have a decision procedure: just check if any strings of length $0, 1, \ldots k-1$ are accepted). You can show this using the pumping lemma!

b) Show that the question, Given a FA $M$ over $\Sigma$, does $M$ accept a string of length $\leq 2$? is decidable

This is a finite set: just check each such string to see if it leads to a final state.