Homework #4 Solutions

#1. Prove that the following is not a regular language: The set of strings of 0's and 1's that are of the form *w w*

<u>Proof</u> by contradiction using the Pumping Lemma

The language is clearly infinite, so there exists *m* (book uses a *k)* such that if I choose a *string* with |*string*| $\geq$ *m,* the 3 properties will hold.

Pick string = $0^m 1 0^m 1$. This has length > *m*.

So, there is an *u*, *v*, *w* such that *string* = *u v w* with |*u v*| $\leq$ m, |*v*| $\geq$ 0.

So uv = 0^n, v = 0^k, and w = $0^{m-n}\ 1\ 0^m\ 1$

And the string *u v v w* is supposed to also be in the language.

But *u v v w* = $0^{n+k}\ 1\ 0^{m-n}\ 1\ 0^m\ 1$, and a few minutes staring at this should convince that there is no way this could be of the form *w w* (Be sure you see why).

#2. Show that the language L = $\{a^p\}$ p is prime is not a regular language

<u>Proof</u> by contradiction using the Pumping Lemma

The language is clearly infinite, so there exists *k* such that if I choose a *string* with |*string*| $\geq$ *k*, the 3 properties will hold.

Pick *string* = 1^p

Then we can break *string* into *u v w* such that *v* is not empty and |*u v*| $\leq$ *k*.
Suppose |v| = *m*. Then |*u w*| = *p – m*.
If the language really is regular, the string $u\, v^{p-m}\, w$ must be in the language.

But, |$u\, v^{p-m}\, w$| = *p – m*+ (*p-m*)*m*
which can be factored into (*m+1*)(*p-m*).
Thus this string does not have a length which is prime, and cannot be in L.
This is a contradiction.

#3. Suppose *h* is the homomorphism from {0,1,2} to {a,b} defined by *h*(0) = a; *h*(1) = ab; *h*(2) = ba.

a) What is *h*(21120)

h(21120) = ba ab ab ba a

b) If L = 01*2, what is *h*(L)?

h(L) = a(ab)*ba

c) If L = a(ba)*, what is h^{-1}(L)?

02*U 1*0

#4. a) Show that the question: *Does L = Σ*?* for regular language L is decidable.

First the question *Does L = Φ is decidable:* Just have to look at the dfa for L to see if there is a path from the start state to a final state.

Now look at the complement of L, L'. It is decidable whether it is empty because the complement of a regular language is regular. If L' is empty, then L =Σ*; otherwise L <> Σ*.

A more interesting proof is that a language is empty (hence its complement is Σ*) if and only if the related dfa accepts a string whose length is less than *k*, the number of states (Then we have a decision procedure: just check if any strings of length 0, 1, … k-1 are accepted). You can show this using the pumping lemma!

b) Show that the question, *Given a FA M over Σ, does M accept a string of length ≤ 2?* is decidable

This is a finite set: just check each such string to see if if it leads to a final state.