CS2022/ MA 2201 Homework #5
Due Tuesday, 4/25
(at the beginning of class)
SOLUTIONS

All questions worth 5 points

#1. Let R be the Congruence Modulo m relation on the integers:
R = {(a,b) | a = b (mod m); that is, m | (a – b) for some positive integer m}.

a) If m = 5, is 2 R -3 ?

Does 5 | 2 – (-3)? I.e., does 5|5? Yes

b) For m = 5, is -3 R 2?

Does 5 | -3 – 2? I.e., does 5|-5? Yes

c) Show the equivalence classes of R for m = 5.

[0] = {…, -10, -5, 0, 5, 10, 15, …}
[1] = {…, -11, -6, 1, 6, 11, 16, …}
[2] = {…, -12, -7, 2, 7, 12, 17, …}
[3] = {…, -13, -8, 3, 8, 13, 18, …}
[4] = {…, -14, -9, 4, 9, 14, 19, …}

#2. Show that Congruence Modulo m is an equivalence relation.

Reflexive

a R a because m | (a – a), i.e., m | 0

Symmetric

If aRb, then m | (a-b), so a| -(a-b) or a | (b-a)

Transitive

If aRb, then m | (a-b) or we can write (a – b) = km, some k
If bRc, then m | (b-c) or we can write (b – c) = lm, some l

Adding these we get (a – c) = (k+l)m, m | (a-c)

#3. a) Show that 55 + 26 = (3 + 2) (mod 4)

a) 55 + 26 = 81; 3 + 2 =5 so we need to show that 81 = 5 (mod 4)
or that 4 | (81 – 5) or 4 | 76 which is true (76 = 4 * 19)

b) Evaluate -17 (mod 2)

-17 = 2(-9) + 1 so -17(mod 2) = 1

c) Evaluate 144 mod 7

144 = 7 *20 + 4 so `144 mod 7 = 4

#4. a) What is the secret message produced from the message "HOW ARE YOU" using the Caesar cipher[1]?

The numeric equivalents are: 8 15 23 1 18 5 25 15 21
Adding 3 (mod 26): 11 18 26 4 21 8 2 18 24
Converting back to letters: KRZ DUH BRX

b) Use the Caesar cipher to decrypt the message L DP ILQH

Converting to numbers: 12 4 16 9 12 17 8
Subtracting 3 (mod 26) 9 1 13 6 9 14 5
Converting back to letters: I AM FINE

#4. a) Encrypt **I LOVE YOU** using the function f(p) = (7p +3) mod 26

I: 7 * 9 + 3 = 66 and 66 mod 26 = 14 → N
L: 7 * 12 + 3 = 87 and 87 mod 26 = 9 → I
O: 7 * 15 + 3 = 108 and 108 mod 26 = 4 → D
V: 7 * 22 + 3 = 157 and 157 mod 26 = 1 → A
E: 7 * 5 + 3 = 38 and 38 mod 26 = 12 → L
Y: 7 * 25 + 3 = 178 and 178 mod 26 = 22 → V
O: 7 * 15 + 3 = 108 and 108 mod 26 = 4 → D
U: 7 * 21 + 3 = 150 and 150 mod 26 = 20 → T

So the encrypted message is: N IDAL VDT

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[1] Caesar's encryption: replace every letter by an integer from 1 to 26; a is 1, b is 2 etc.; Then apply the function f(p) = (p + 3) mod 26 and convert the numbers back to the appropriate letter. Thus "j" is converted to 10, then to 13 and back to "M". ($f^{-1}(p) = (p – k) \bmod 26$)

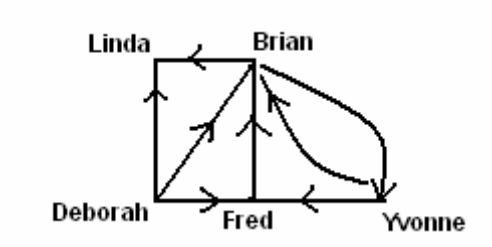
#5. Books are identified by an International Standard Book Number (ISBN), a 10-digit code, x1, x2, …x10, assigned by the publisher. These 10 digits consist of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally, 1 1-digit check digit that is either a digit or the letter X (representing 10). This digit is selected so that

$$\sum_{i=1}^{10} i x_i = 0 \pmod{11}$$

and is used to detect errors in individual digits and transposition of digits. If the ISBN of a book is 0-201-57Q89-1, where Q is a digit, find Q.

1*0 + 2*2 + 3*0 + 4*1 + 5*5 + 6*7 + 7*Q + 8*8 + 9*9 + 10*1 = 0 (mod 11)
That is, 230 + 7Q = 0 (mod 11)
Subtracting 230 from both sides
7Q = 1 (mod 11) because 231 = 11*21
Trial and error gives Q = 8 (because 56 = 1 (mod 11))
(Look at [1] = {…1, 12, 23, 34, 45, 56, …} and note that 56 is the first multiple of 7)

#7. Section 8.1, #2. Consider the following influence graph (example 3 in the text).



a) Identify the set of vertices and edges:

Vertices:

{Linda, Brian, Deborah, Fred, Yvonne}

Edges:

(Deborah, Linda)
(Deborah, Fred)
(Deborah, Brian)
(Fred, Brian)
(Brian, Linda)
(Brian, Yvonne)

(Yvonne, Brian)

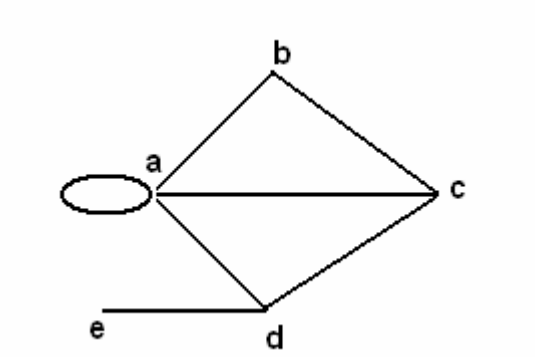
b) Who influences Fred and who can Fred influence?

Fred is influenced by Deborah and Yvonne
Fred influences Brian

#8. Draw a graph with the specified properties or explain why no such graph can exist

a) Graph with five vertices of degrees 1, 2, 3, 3, and 5

Here's one:



b) Graph with four vertices of degrees 1, 2, 3 and 3

Not possible