

CS2022/ MA 2201 Homework #5
Due Tuesday, 4/25
(at the beginning of class)

This homework is the sole work of: _____ **whose conference section is at:**

_____ **whose conference section is at:**

Sources (People, URL's, Books etc.) consulted:

Source _____ **for Problem #** _____

Source _____ **for Problem #** _____

Source _____ **for Problem #** _____

Date: _____

Reference (if any):

All questions worth 5 points

#1. Let R be the Congruence Modulo m relation on the integers:

$R = \{(a,b) \mid a = b \pmod{m}\}$; that is, $m \mid (a - b)$ for some positive integer m .

a) If $m = 5$, is $2 R -3$?

b) For $m = 5$, is $-3 R 2$?

c) Show the equivalence classes of R for $m = 5$.

#2. Show that Congruence Modulo m is an equivalence relation.

Reflexive

Symmetric

Transitive

#3. a) Show that $55 + 26 = (3 + 2) \pmod{4}$. **Show your work**

a) **Evaluate $-17 \pmod{2}$**

b) **Evaluate $144 \pmod{7}$**

#4. a) What is the secret message produced from the message “HOW ARE YOU” using the Caesar cipher¹?

b) **Use the Caesar cipher to decrypt the message L DP ILQH Show your work**

#5. a) Encrypt **I LOVE YOU** using the function $f(p) = (7p + 3) \pmod{26}$ **Show your work**

#6. Books are identified by an International Standard Book Number (ISBN), a 10-digit code, x_1, x_2, \dots, x_{10} , assigned by the publisher. These 10 digits consist of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally, 1 1-digit check digit that is either a digit or the letter X (representing 10). This digit is selected so that

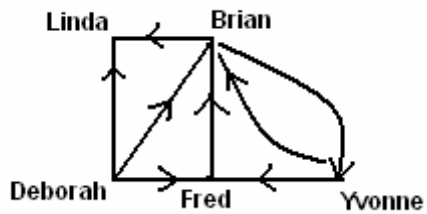
$$\sum_{i=1}^{10} ix_i = 0 \pmod{11}$$

and is used to detect errors in individual digits and transposition of digits.

If the ISBN of a book is **0-201-57Q89-1**, where Q is a digit, find Q. **Show your work**

#7. Section 8.1, #2. Consider the following influence graph (example 3 in the text).

¹ Caesar’s encryption: replace every letter by an integer from 1 to 26; a is 1, b is 2 etc.; Then apply the function $f(p) = (p + 3) \pmod{26}$ and convert the numbers back to the appropriate letter. Thus “j” is converted to 10, then to 13 and back to “M”. ($f^{-1}(p) = (p - k) \pmod{26}$)



a) Identify the set of vertices and edges:

Vertices:

Edges:

b) Who influences Fred and who can Fred influence?

#8. Draw a graph with the specified properties or explain why no such graph can exist

- a) Graph with five vertices of degrees 1, 2, 3, 3, and 5
- b) Graph with four vertices of degrees 1, 2, 3 and 3
- c) Graph with four vertices of degrees 1, 2, 3 and 5