Homework #3
Due Tuesday 4/11
(at the beginning of class)

This homework is the sole work of: ________________ whose conference section is at:

________________ whose conference section is at:

Sources (People, URL's, Books etc.) consulted:

Source _______ for Problem # ________

Source _______ for Problem # ________

Source _______ for Problem # ________

Date: __________

#1. (5 Points) Section 3.4, #48 a,b. Find these values of Ackerman's Function

a) A(1,0)
d) A(2,2)

Ackerman's function, A(m,n) is defined:

$$A(m,n) = \begin{cases} 2n & \text{if } m = 0 \\ 0 & \text{if } m \geq 1 \text{ and } n = 0 \\ 2 & \text{if } m \geq 1 \text{ and } n = 1 \\ A(m-1, A(m,n-1)) & \text{if } m \geq 1 \text{ and } n \geq 2 \end{cases}$$

Solution

a) A(1,0) = 0 by the 2nd line of the definition
b) A(2,2) = A(1, A(2,1)) = A(1,2) = A(0, A(1,1)) = A(0,2) = 4

#2. (5 Points) Section 4.1, #32. How many functions are there from the set {1, 2, 3, …,n} (when n is a positive integer) to the set {0,1}.

There are 2^n such functions, since there is a choice of two function values for each of the n elements of the domain.

#3. A computer password consists of from 1 to 3 letters chosen from the 26 letters in the alphabet (repetitions allowed). How many passwords are possible?

Solution

Partition the passwords into ones of length 1, of length 2 and of length 3 and use the Rule of Sums:

Use the rule of products to get each of the three:

# of passwords of length 1 = 26
# of passwords of length 2 = 26^2
# of passwords of length 3 = $\underline{26^3}$
Total 18,278

#4. Show that if *f* is a function from S → T where S and T are finite sets with $|S| > |T|$, then there are elements s_1 and s_2 with

$$f(s_1) = f(s_2)$$

i.e., f is not 1-1

True by the Pigeonhole Principle, where there are |T| Pigeonholes and |S| Pigeons. Since $|S| > |T|$, there are more Pigeons (elements of S) than Pigeonholes (elements of T), so more than 1 element in S must map to the same element (Pigeonhole) in T.

#5. Section 4.2, #18 Suppose there are 9 students in a discrete math class (can't be WPI!).

a) **Show that the class must have at least 5 male students or at least 5 female students.**

If not, then there would be 4 or fewer male students and 4 or fewer female students for a total of 8. But there are 9 students

b) **Show that the class must have at least 3 male students or at least 7 female students**

If not, then there would be 2 or fewer males and 6 or fewer females for a total of 8, and, once again, there are 9 students.

#6.

Let S = {1, 2, 3, 4, 5}

a) **How many 3-permutations of S are there? List 5 of them.**

P(5,3) = 5!/(5-3)! = 5!/2! = 5 * 4 * 3 * 2 * 1 / 2 * 1 = 60

123 132 213 231 312 etc.

b) **How many 3-combinations of S are there? List 5 of them.**

C(5,3) = 5!/ 3! *(5-3)! = 5!/3! * 2! = 5 * 4 * 3 * 2 * 1 / 3 * 2 * 1 * 2 * 1 = 10
123 134 234 245 etc.

#7. How many ways can 3 of the letters in the word ALGORITHM be selected and written in a row?

Solution

$$P(9,3) = \frac{9 * 8 * 7 * 6!}{6!} = 504$$

#8. How many 8-bit binary strings have exactly three 1's?

$$C(8,3) = \frac{8!}{3! * 5!} = \frac{8 * 7 * 6 * 5!}{3 * 2 * 5!} = 56$$