

**Homework #3**  
**Due Friday, 4/14**  
**(at the beginning of class)**

This homework is the sole work of: \_\_\_\_\_ whose conference section is at:  
\_\_\_\_\_ whose conference section is at:

Sources (People, URL's, Books etc.) consulted:

Source \_\_\_\_\_ for Problem # \_\_\_\_\_

Source \_\_\_\_\_ for Problem # \_\_\_\_\_

Source \_\_\_\_\_ for Problem # \_\_\_\_\_

Date: \_\_\_\_\_

#1. (2 Points) Play with the domino presentation at:  
[http://web.cs.wpi.edu/~kal/courses/cs503/module0/domino\\_part1.html](http://web.cs.wpi.edu/~kal/courses/cs503/module0/domino_part1.html) and **and**  
[http://web.cs.wpi.edu/~kal/courses/cs503/module0/domino\\_part2.html](http://web.cs.wpi.edu/~kal/courses/cs503/module0/domino_part2.html) and (for the 2  
points) tell us whether it helped you and why or why not.

#2. (6 points) Section 2.2, #2: Determine whether each of these functions is  $O(x^2)$ .

- a)  $f(x)=17x+11$
- b)  $f(x)=x^2+1000$
- c)  $f(x)=x \log x$
- d)  $f(x)=x^4/2$
- e)  $f(x)=2^x$
- f)  $f(x)=\lfloor x \rfloor * \lceil x \rceil$

#3. (12 Points) Section 2.2 #22 (Hint: it may help to check the answers to #1 in the library first)

For each function in Exercise 1, determine whether that function is  $\Omega(x)$  and whether it is  $\Theta(x)$ .

Functions from Exercise 1, Section 2.2:

- a)  $f(x)=10$
- b)  $f(x)=3x + 7$
- c)  $f(x)=x^2 + x + 1$
- d)  $f(x)=5 \log x$
- e)  $f(x)=\lfloor x \rfloor$
- f)  $f(x)=\lceil x/2 \rceil$

#4. (4 Points) Section 2.3, #14 An algorithm is called **optimal** for the solution of a problem with respect to a specified operation if there is no algorithm for solving this problem using fewer operations.

a) Show that Algorithm 1 in Section 2.1 is an optimal algorithm with respect to the number of comparisons of integers. (Note: Comparisons used for bookkeeping in the loop are not of concern here.)

*Algorithm 1 from section 2.1: Finding the Maximum Element in a Finite Sequence*

```
procedure max ( $a_1, a_2, \dots, a_n$ : integers)
  max :=  $a_1$ 
  for k:=2 to n
    if max <  $a_k$  then max :=  $a_k$ 
  {max is the largest element}
```

b) Is the linear search algorithm optimal with respect to the number of comparisons of integers (not including comparisons used for bookkeeping in the loop)?

*Linear Search Algorithm*

```
procedure linear search (x: integer,  $a_1, a_2, \dots, a_n$ : distinct integers)
  k:=1

  while (k <= n and x <>  $a_k$ )
    k:=k+1
  if k <= n then location := k
  else location := 0
  {location is the subscript of the term that equals x, or is 0 if x is not found}
```

#5. (4 Points) Section 3.3, #12 Prove (using induction) that  $3^n < n!$  (n factorial) whenever n is a positive integer greater than 6.

#6. (4 Points) Section 4.1, #44 Every student in a discrete mathematics class is either a CS or a Math or is a joint major in these two subjects. How many students are in the class if there are 38 CS majors (including joint majors), 23 Math majors (including joint majors) and 7 joint majors?

#7. (4 Points) Section 4.3, #14 In how many ways can a set of two positive integers less than 100 be chosen?

#8. (4 Points) Section 5.1, #8 What is the probability that a five-card poker hand contains the ace of hearts?