Name____________________

MA2201 / CS2022
Test #1

Tuesday, April 4, 2006

#1.
a) (4 Points) Given

p: Today is Tuesday
q: We have a test today

Write propositions for each of the following:

(i) If today is Tuesday, then we have a test today

$$p \rightarrow q$$

(ii) If we don't have a test today, then today is not Tuesday

$$\sim q \rightarrow \sim p$$

(iii) If we have a test today, then today is Tuesday

$$q \rightarrow p$$

(iv) If today is not Tuesday, then we do not have a test today.

$$\sim p \rightarrow \sim q$$

b) (2 Points) Given:

In(s,m): Student *s* is enrolled in math class *m*
C(s): Student *s* owns a cat

Write the predicates for:

(i) There is at least one math student (that is, some student in some math class) who owns a cat.

$$\exists s\ \exists m\ [\ In(s,m) \wedge C(s)\ ]$$

(ii) No math student owns a cat.

$$\forall s\ \forall m \sim [In(s,m) \wedge C(s)]$$

#2. (4 Points) For sets A and B, show A x B when:

(i) $A = \{1, 2\}, B = \varnothing$

$\varnothing$

(ii) $A = \{1, 2\}, B = \{\varnothing\}$

$\{(1, \varnothing), (2, \varnothing)\}$

#3. For sets A and B, show $\sim(A \cap B) = \sim A \cup \sim B$:

a) (8 Points) with a proof showing each side is a subset of the other. (Be clear!)

You need to show $\sim(A \cap B) \subseteq \sim A \cup \sim B$ and $\sim A \cup \sim B \subseteq \sim(A \cap B)$

1. $\underline{\sim(A \cap B) \subset \sim A \cup \sim B}$

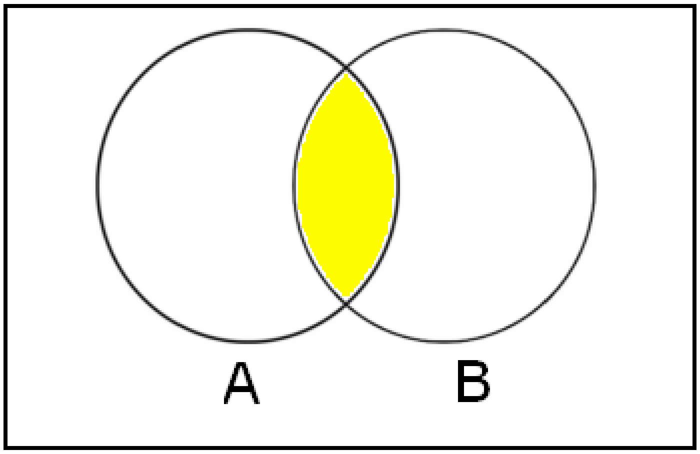
Suppose $x \varepsilon \sim(A \cap B)$	**Then $x \notin A \cap B$**	**Definition of Complement**
	Thus $x \notin A$ or $x \notin B$	**Definition of Intersection**
	And $x \varepsilon \sim A$ or $x \varepsilon \sim B$	**Definition of Complement**
	Thus $x \varepsilon \sim A \cup \sim B$	**Definition of Union**

2. $\sim A \cup \sim B \subseteq \sim(A \cap B)$

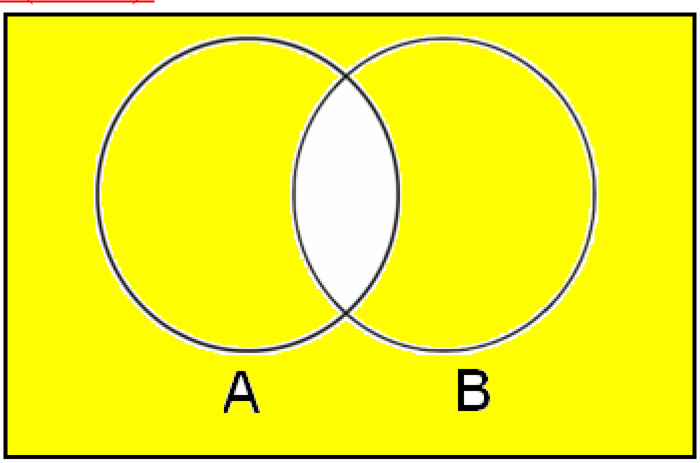
Suppose $x \varepsilon \sim A \cup \sim B$	**Then $x \varepsilon \sim A$ or $x \varepsilon \sim B$**	**Definition of Union**
	Thus $x \notin A$ or $x \notin B$	**Definition of Complement**
	And $x \notin A \cap B$	**Definition of Intersection (if it's not in 1 of A or B, it cannot be in the intersection)**
	Thus $x \varepsilon \sim(A \cap B)$	

(4 Points) with Venn diagrams

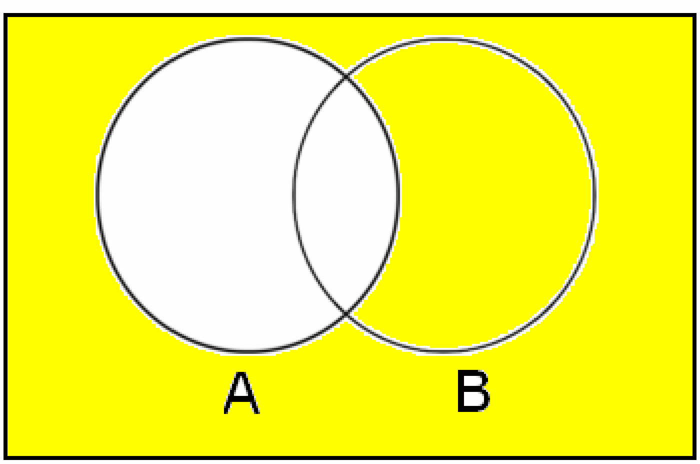
A ∩ B:



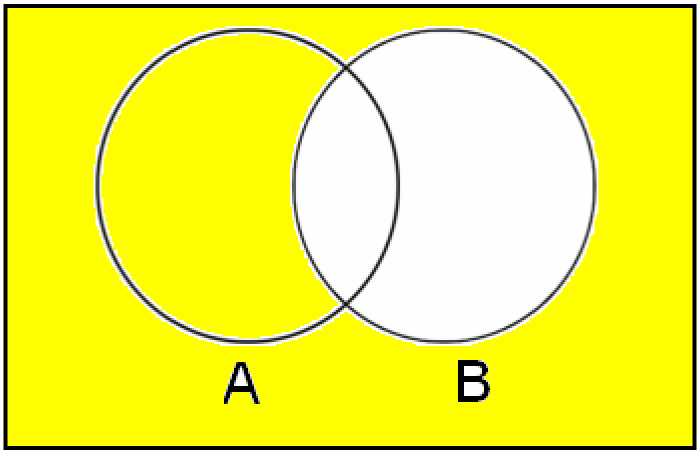
~ (A ∩ B):



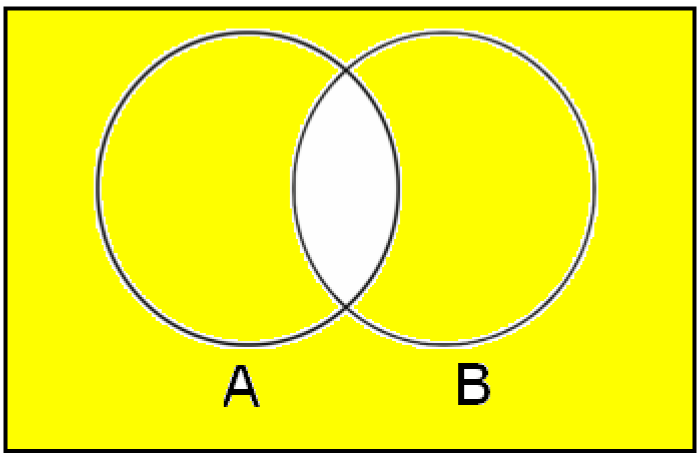
~ A:



~ B:



~A ∪ ~ B:



Note that the diagram for ~A ∪ ~ B is the same as the diagram for ~ (A ∩ B):

#4 (6 Points) If $f(x) = \sum_{i=1}^{5} x^i$, what is O(f(x))?

$$\sum_{i=1}^{5} x^i = x^1 + x^2 + x^3 + x^4 + x^5$$

That is $f(x) = x^5 + x^4 + x^3 + x^2 + x^1$
and we know that the Big-O of a polynomial is its degree
So $O(f(x) = O(x^5)$

Alternatively, you can find a C and a k such than $f(x) \leq C\, x^n$ for $x > k$:

$x^5 + x^4 + x^3 + x^2 + x^1 \leq x^n + x^n + x^n + x^n + x^n = 5\, x^n$
So C= 5, k = 1 work fine (as do other values)

#5. (6 Points) Determine if the following are True or False. If true, prove it; if false, state a counter-example.

a) $f(x) = \lfloor x \rfloor$ is a 1-1 where x is a real number

$\lfloor x \rfloor$: Reals → Integers and both 1.1 and 1.2 map to 1, so it is not 1-1

b) $f(x) = \lfloor x \rfloor$ is onto where x is a real number

For any integer z, $\lfloor z \rfloor \rightarrow z$ so $\lfloor\ \rfloor$ is onto

c) The sum of any two even integers is even

Basically, this says that if 2 integers, say m and n are even, then their sum is even. So: m even, n even, are the given and we want to show m + n is even.

Proof

m is even	**Premise (or Given)**
so $m = 2k_1$	**Definition of even**
n is even	**Premise (or Given)**
so $n = 2k_2$	**Definition of even**
$m + n = 2k_1 + 2k_2$	**Arithmetic**
$= 2(k_1 + k_2)$	**Arithmetic**
So m + n is even	**Definition of even**

#6. (6 Points) Prove by mathematical induction

$$2n + 1 < 2^n$$

Basis

This isn't true until n = 3:

$$2n+1 = 2 * 3 + 1 = 7$$

$$2^n = 2^3 = 8$$

Inductive Step

Assume $2n + 1 < 2^n$ for $n \geq 3$ and show $2(n+1) + 1 < 2^{n+1}$:

$2(n+1) + 1 = 2n + 1 + 2$	**Arithmetic**
$< 2^n + 2$	**I.H.**
$< 2^{n+1}$	**Arithmetic***

*** $2^n + 2 < 2^{n+1} \Leftrightarrow 2 < 2^{n+1} - 2^n = 2^n(2-1) = 2^n$ which is true for $n \geq 2$**