Homework #2
Due Tuesday, 3/28
(at the beginning of class)

This homework is the sole work of: ________________ whose conference section is at:

________________ whose conference section is at:

Sources (People, URL's, Books etc.) consulted:

Source _______ for Problem # ________

Source _______ for Problem # ________

Source _______ for Problem # ________

Date: ___________

Each question is worth 5 Points

#1. Page 75, #20. **Prove the square of an even number is an even number**

We want to show that if n is even then n^2 is even. Many of you misread this!

Proof

Suppose that n is even.

Then $n = 2k$ for some integer k	Definition of even
Then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$	Arithmetic
Therefore n^2 is even	Definition of even

2. a) Section 1.6, page 85, #12: **Find 2 sets A and B such that A ε B and A $\subseteq$ B**

Hmmm. For A to have those 2 properties, A must itself be a set and B must contain that set. The simplest example would be A = Ø and B = {Ø}

But any set A that you put inside set B will work.

b) Section 1.6, page 85, #14 **What is the cardinality of each of the following sets?**

a) Ø 0

b) {Ø} 1

c) {Ø,{Ø}} 2

d) {Ø,{Ø}, {Ø,{Ø}}} 3

#3. a) Section 1.6, page 85, #16: **Can you conclude that A = B if A and B are 2 sets with the same power set? Why or why not?**

The union of all the sets in the power set of X is X, so we can recover a set from its power set. The answer is "yes"

b) Section 1.6, page 86, #22: **Suppose that A x B = Ø, where A and B are sets. What can you conclude?**

One of A or B (or both) must be empty (if neither A nor B were empty, there would be an element in AxB)

#4. a) Section 1.7, page 95, #14a,e: **Let A, B and C be sets. Show that**

a) (A U B) ⊆ (A U B U C)

(i) in words by showing the appropriate subset relations as done in class

Suppose x ε A U B
Then x ε A or ε B
Therefore x ε A U B U C
(truthfully, this is almost given to be true by the definition of union)

b) (B – A) U (C – A) = (B U C) – A

We need to show:

1. (B – A) U (C – A) ⊆ (B U C) – A
and 2. (B U C) – A ⊆ (B – A) U (C – A)

<u>1. (B – A) U (C – A) ⊂ (B U C) – A</u>

If x ε (B – A) U (C – A), then x ε B – A or x ε C – A

<u>Case 1</u> x ε B – A

Then x ε B so x ε B U C
And x is not in A
So x ε ((B U C) – A

<u>Case 2</u> x ε C – A

Then x ε C so x ε B U C
And x is not in A

So, again, x ε ((B ∪ C) – A

2. (B ∪ C) – A ⊆ (B – A) ∪ (C – A)

x ε (B ∪ C) – A
Then x ε B or x ε C AND x is not in A

<u>Case 1</u>

x ε B AND x is not in A
Then x ε B – A
So x ε (B – A) U (C – A)

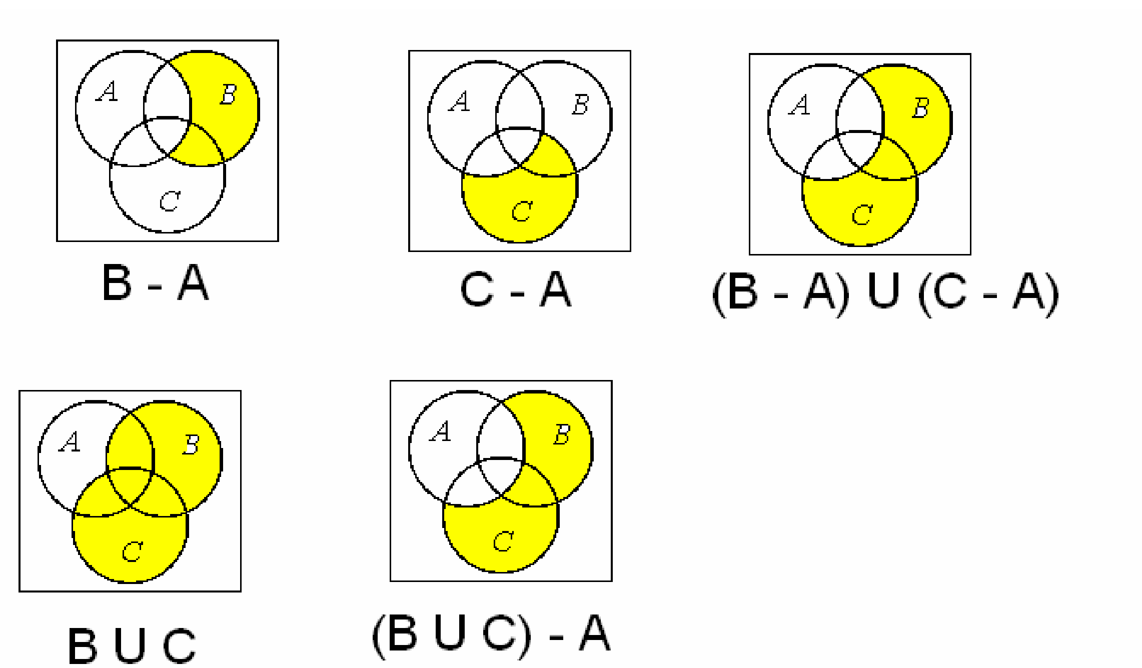
<u>Case 2</u>

x ε C AND x is not in A
Then x ε C – A
So x ε (B – A) U (C – A)

∴ (B ∪ C) – A ⊆ (B – A) ∪ (C – A)

We have shown (B – A) ∪ (C – A) = (B ∪ C) – A

(ii) using Venn diagrams



#5. a) Determine whether each of these functions is a bijection**. If it is a bijection, prove it; if not a bijection, give a counterexample.**

A bijection is a 1-1 and onto function

a) f: R → Z: f(x) = |_x_| (that is, f(x) = floor(x))

1-1

Not 1-1. For example floor(1.1) = floor(1) = a

onto

It is onto: Given any integer n, floor(n) = n (among others)

b) f: R → R: f(x) = x^2

1-1

Not 1-1. For example -1 → 1 and 1 → 1

onto

It is onto. For any x in the codomain, sqrt(x) → x

c) f: {1,2,3} → {2}: {(1,2). (2,2), (3,2)}

1-1

Not 1-1: All numbers map to 2

Onto

It is onto since {2} is the entire codomain and at least (actually 3) number in the domain maps to 2.

d) f: {1,2,3} → {1,2, 3}: {(1,2), (1,3), (2,1), (2,2)}

Not even a function! 1 maps to both 2 and 3 (and 2 maps to both 1 and 2).

b) **Let f be a function from the set A to the set B. Let S and T be subsets of A, R a subset of B. Show that:**

a) f(S U T) = f(S) U f(T)

If y ε f(SUT), then ∃ x ε SUT such that f(x) = y

Then x ε S or x ε T.

Case 1

x ε S

Then f(x) ε f(S)

So f(x) ε f(S) U f(T)

Case 2

x ε T
Then f(x) ε f(T)
So f(x) ε f(S) U f(T)

b) $f(f^{-1}(R)) \subseteq R$

Suppose y ε $f(f^{-1}(R))$. Then ∃ x ε $f^{-1}(R))$ such that f(x) = y
If x ε $f^{-1}(R))$ then f(x) ε R (by definition)

Thus, $f(f^{-1}(R)) \subseteq R$

#6. Show that 5^n is $O(6^n)$, but 6^n is not $O(5^n)$

Since $5^n \leq 6^n$ for all n > 0, 5^n is $O(6^n)$ (C = 1, k = 0).

If 6^n were $O(5^n)$, then for some C, $6^n \leq C5^n$, for sufficiently large n.

That is $C \geq (6/5)^n$ for all sufficiently large n, which is impossible.

#7. Section 2.2, page 142, #16 **Show that if f(x) is $O(x^2)$, then f(x) is $O(x^3)$**

if f(x) is $O(x^2)$, then $|f(x)| \leq C x^2$ for all x > k, some C.
Let k' = max (k,1).
$|x^2| \leq |x^3|$ for all $x \geq 1$, so $|f(x)| \leq C |x^3|$ for all x > k'.

#8. Express each statement using Ω-, O- or Θ- notation

a) Show that $2^x + 17$ is $O(3^x)$.

If x > 5, $2^x + 17 \leq 2^x + 2^x = 2 * 2^x \leq 2 * 3^x$ so $2^x + 17$ is $O(3^x)$ (C = 2, k = 5)

b) Show that $(x^3 + 2x)/(2x+1)$ is $O(x^2)$

$(x^3 + 2x)/(2x+1) \leq (x^3 + 2x^3)/(2x) = 3/2\ x^2$
So, $(x^3 + 2x)/(2x+1)$ is $O(x^2)$ with k = 1, C = 3/2

c) Show that $2x^2 + x - 7$ is $\Theta(x^2)$

For large x, $x^2 \leq 2x^2 + x - 7$
For $x \geq 1$, $2x^2 + x - 7 \leq 3x^2$

d) Show that floor(x + ½) is Θ(x)

For $x > 2$, floor $(x + ½) \leq 2x$ and also $x \leq 2$ floor$(x + ½)$

e) Show that ceiling(xy) is Ω (xy)

For all positive values of x and y, ceiling$(xy) \geq xy$.
Thus, ceiling(xy) = Ω(xy) (with $C = 1$ and $k_1 = k_2 = 0$)