

Homework #2
Due Tuesday, 3/28
(at the beginning of class)

This homework is the sole work of: _____ whose conference section is at:
_____ whose conference section is at:

Sources (People, URL's, Books etc.) consulted:

Source _____ for Problem # _____

Source _____ for Problem # _____

Source _____ for Problem # _____

Date: _____

Each question is worth 5 Points

#1. Page 75, #18. **Prove that the square of an even number is an even number.**

#2. a) Section 1.6, page 85, #12: **Find 2 sets A and B such that $A \in B$ and $A \subseteq B$**

b) Section 1.6, page 85, #14 **What is the cardinality of each of the following sets?**

- a) \emptyset
- b) $\{\emptyset\}$
- c) $\{\emptyset, \{\emptyset\}\}$
- d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

#3. a) Section 1.6, page 85, #16: **Can you conclude that $A = B$ if A and B are 2 sets with the same power set? Why or why not?**

b) Section 1.6, page 86, #22: **Suppose that $A \times B = \emptyset$, where A and B are sets. What can you conclude?**

#4. a) Section 1.7, page 95, #14a,e: **Let A, B and C be sets. Show that**

- a) $(A \cup B) \subseteq (A \cup B \cup C)$
- b) $(B - A) \cup (C - A) = (B \cup C) - A$

- (i) in words by showing the appropriate subset relations as done in class
- (ii) using Venn diagrams

#5. a) Determine whether each of these functions is a bijection. **If it is a bijection, prove it; if not a bijection, give a counterexample.**

- a) $f: \mathbb{R} \rightarrow \mathbb{Z}: f(x) = \lfloor x \rfloor$ (that is, $f(x) = \text{floor}(x)$)
- b) $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = x^2$
- c) $f: \{1,2,3\} \rightarrow \{2\}: \{(1,2), (2,2), (3,2)\}$
- d) $f: \{1,2,3\} \rightarrow \{1,2,3\}: \{(1,2), (1,3), (2,1), (2,2)\}$

b) Let f be a function from the set A to the set B . Let S and T be subsets of A , R a subset of B . Show that:

- a) $f(S \cup T) = f(S) \cup f(T)$
- b) $f(f^{-1}(R)) \subseteq R$

#6. Show that 5^n is $O(6^n)$, but 6^n is not $O(5^n)$

#7. Section 2.2, page 142, #16 Show that if $f(x)$ is $O(x^2)$, then $f(x)$ is $O(x^3)$

#8. Express each statement using Ω -, O - or Θ - notation

- a) Show that $2^x + 17$ is $O(3^x)$.
- b) Show that $(x^3 + 2x)/(2x+1)$ is $O(x)$
- c) Show that $2x^2 + x - 7$ is $\Theta(x^2)$
- d) Show that $\text{floor}(x + 1/2)$ is $\Theta(x)$
- e) Show that $\text{ceiling}(xy)$ is $\Omega(xy)$