

Homework #1 Solutions

Each question is worth 5 Points

#1. Construct truth tables for

i) $p \vee q \rightarrow r$

ii) $(p \rightarrow r) \wedge (q \rightarrow r)$

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$p \vee q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

b) What is your conclusion about these? **They are equivalent**

#2. **Page 16, #10:** Let p , q , and r be the propositions

p : You get an A on the final exam

q : You do every exercise in this book

r : You get an A in the class

Write these propositions using p , q , and r and the Boolean connectives

- a) You get an A in this class, but you do not do every exercise in the book
- b) You get an A on the final, you do every exercise in this book, and you get an A in this class
- c) To get an A in this class, it is necessary for you to get an A on the final
- d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class
- e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
- f) You get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Using p , q , r above express each of the following as an English sentence

- g) $p \leftrightarrow r$
- h) $p \vee q \rightarrow r$
- i) $\sim r \rightarrow \sim q$
- j) $\sim r \wedge q \rightarrow \sim p$

- a) $r \wedge \sim q$ b) $p \wedge q \wedge r$ c) $r \rightarrow p$ d) $p \wedge \sim q \wedge r$ e) $(p \wedge q) \rightarrow r$
- f) $r \leftrightarrow (q \vee p)$
- g) You get an A in the course if and only if you get an A on the final
- h) If you get an A on the final or you do every exercise in the book, then you get an A in the course
- i) If you don't get an A in the course and don't do every exercise, you won't get an A on the final
- j) If you don't get an A in the course and you don't do all the exercises you won't get an A on the final.

#3. Page 17, #14

For each of these sentences, determine whether an inclusive or an exclusive or is intended

- a) The employer making this request would be happy if the applicant knew both of these languages, so this is clearly an inclusive or.
- b) The restaurant would probably charge extra if the diner wanted both of these items, so this is an exclusive or
- c) If a person happened to have both forms of identification, so much the better, so this is clearly an inclusive or
- d) This could be argued either way, but the inclusive interpretation seems more appropriate. This phrase means that faculty members who do not publish paper in research journals are likely to be fired from their jobs during the probationary period. On the other hand, it may happen that they will be fired even if they do publish (for example, if their teaching is poor).

#4. Page 40, #10 Let:

C(x): x has a cat

D(x): x has a dog

F(x): x has a ferret

a) A student in your class has a cat, a dog, and a ferret: **We assume that this means that one student has all three animals:**

$\exists x (C(x) \wedge D(x) \wedge F(x))$

b) All students in your class have a cat, a dog, and a ferret: $\forall x (C(x) \wedge D(x) \wedge F(x))$

c) Some student in your class has a cat and a ferret, but not a dog: $\exists x (C(x) \wedge F(x) \wedge \sim D(x))$

d) No student in your class has a cat, a dog and a ferret: This is the negation of part a): $\neg \exists x (C(x) \wedge D(x) \wedge F(x))$

e) For each of the 3 animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet: Here the owners of these pets can be different: $(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$. There is no harm in using the same dummy variable because the scopes are different, but this could also be written, for example, as $(\exists x C(x)) \wedge (\exists y D(y)) \wedge (\exists z F(z))$

#5. Page 53, #14. Use quantifiers and predicates with more than 1 variable to express these statements

The answers to this exercise are not unique; there many ways of expressing the same propositions symbolically. If you think your answer is correct, see the TA who graded it. Our universe of discourse for persons here consists of people in this class. We need to make up a predicate in each case.

a) **There is a student in this class who can speak Hindi:**

**Let $S(x, y)$ mean that person x can speak language y .
Then our predicate is $(\exists x) S(x, \text{Hindi})$.**

b) **Everyone plays some sport**

**Let $P(x, y)$ mean that person x plays sport y .
Then our predicate is $(\forall x) (\exists y P(x, y))$.**

c) **Some student in this class has visited Alaska, but has not visited Hawaii**

**Let $V(x, y)$ mean that person x has visited state y .
Then our predicate is $(\exists x) (V(x, \text{Alaska}) \wedge \neg V(x, \text{Hawaii}))$.**

d) **All students in this class have learned at least one programming language**

**Let $L(x, y)$ mean that person x has learned programming language y .
Then our predicate is $(\forall x) (\exists y L(x, y))$.**

e) **There is a student who has taken every course offered by one of the departments in this school**

Let $T(x, y)$ mean that person x has taken course y , and let $O(y, z)$ mean that course y is offered by department z .

Then our predicate is $(\exists x)(\exists z) (\forall y(O(y,z)\rightarrow T(x, y)))$.
 This could have more or fewer parentheses for clarity; e.g.,
 $(\exists x)((\exists z) (\forall y(O(y,z)\rightarrow T(x, y))))$.
 After awhile, the parentheses, themselves, make it less clear

f) Some student in this class grew up in the same town as exactly one other student in this class:

Let $G(x, y)$ mean that persons x and y grew up in the same town.
 Then our predicate is $(\exists x) (\exists y) (x \neq y \wedge G(x, y) \wedge \forall z (G(x, z) \rightarrow (x=y) \vee (x=z)))$

#6. a) Page 73, #4

Construct an argument using rules of inference to show that the hypotheses “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on”, “If the sailing race is held, then the trophy will be awarded,” and “The trophy was not awarded” imply the conclusion “It rained”.

Let r be the proposition “it rains,”
 Let f be the proposition “It is foggy,”
 Let s be the proposition “The sailing race will be held,”
 Let l be the proposition “The life saving demonstration will go on,” and
 Let t be the proposition “The trophy will be awarded.”

We are given premises $(\neg r \vee \neg f) \rightarrow (s \wedge l)$, $s \rightarrow t$, and $\neg t$. We want to conclude r .
 We set up the proof in two columns, with reasons, as in Example 6, and as done in class. Note that it is valid to replace sub-expressions by other expressions logically equivalent to them.

Step	Reason
1. $\neg t$	Hypothesis
2. $s \rightarrow t$	Hypothesis
3. $\neg s$	Modus tollens using Steps 1 and 2
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Hypothesis
5. $(\neg (s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of Step 4
6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$	De Morgan’s law and double negative
7. $(\neg s \vee \neg l)$	Addition, using Step 3
8. $(r \wedge f)$	Modus ponens using Steps 6 and 7
9. r	Simplification using Step 8

b) Page 74, #12 For each of these arguments, determine whether the argument is correct or incorrect and explain why

a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university:

This is correct, using universal instantiation and modus tollens:

Let $E(x)$ denote “ x is enrolled in the university,” let $D(x)$ denote “ x has lived in a dormitory”

Step	Reason
1. $\forall x (E(x) \rightarrow D(x))$	Premise
2. $E(\text{Mia}) \rightarrow D(\text{Mia})$	Universal instantiation
3. $\neg D(\text{Mia})$	Premise
4. $\neg E(\text{Mia})$	Modus tollens using Steps 2 and 3

b) A convertible car is fun to drive. Isaac’s car is not a convertible. Therefore, Isaac’s car is not fun to drive ($\sim f$).

This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.

Let $C(x)$ denote “ x is a convertible car,” let $D(x)$ denote “ x is fun to drive”. Then the premise is

$\forall x (C(x) \rightarrow D(x))$.

By applying universal instantiation, we get

$C(\text{Isaac’s car}) \rightarrow D(\text{Isaac’s car})$.

Given the premise

“ $\neg C(\text{Isaac’s car})$ ”, this argument is of the form:

$[(p \rightarrow q) \wedge \neg p] \rightarrow \neg q$. This uses the fallacy of denying the hypothesis.

c) Quincy likes all action movies. Quincy likes the movie *Eight Men Out*.

After applying universal instantiation, it contains the fallacy of affirming the conclusion.

Let $A(x)$ denote “ x is an action movie,” let $Q(x)$ denote “Quincy likes movie x ”. Then the premise is

$\forall x (A(x) \rightarrow Q(x))$. By applying universal instantiation, we get

$A(\text{Eight Men Out}) \rightarrow Q(\text{Eight Men Out})$.

Given the premise

“ $Q(\text{Eight Men Out})$ ”, this argument is of the form

$[(p \rightarrow q) \wedge q] \rightarrow p$. This uses the fallacy of affirming the conclusion.

d) All lobstermen (l) set at least a dozen traps (t). Hamilton is a lobsterman. Therefore Hamilton sets at least a dozen traps. This is correct, using universal instantiation and modus ponens.

Step	Reason
1. $\forall x (L(x) \rightarrow S(x))$	Premise
2. $L(\text{Hamilton}) \rightarrow S(\text{Hamilton})$	Universal instantiation
3. $L(\text{Hamilton})$	Premise
4. $S(\text{Hamilton})$	Modus ponens

#7. Prove or disprove: If $1 + 1 = 3$, then $2 + 2 = 7$

The implication is True. An implication is True when the hypothesis is False, no matter what the truth value of the conclusion is. Since $1 + 1 = 3$ is False, the implication is True.

#8. Given that an integer n is even if there is an integer i such that $n = 2 * i$ and an integer n is odd if there is an integer i such that $n = 2 * i + 1$, prove that for every integer $n \geq 0$, n is cannot be both even and odd.

Part 1 (n cannot be both even and odd)
Assume n is both even and odd for $n \geq 0$
That is, $n=2i$ and $n=2j+1$
Case 1 If $i=j$
$2i = 2i+1$
$0 = 1$ (subtracting $2i$)
Therefore i cannot equal j
Case 2 If $i <> j$
$2i = 2j+1$
$i = j+1/2$ by dividing by 2
Cannot be true since i,j are integers; therefore, n cannot be even and odd at the same time

Note that you cannot start out with just “assume $2i = 2i + 1$ ” because you don’t know if i and j are equal (it’s only 1 case).