

## Lecture 19: Boolean Algebra

- Basic Boolean Algebra
- Boolean Functions in Assembly

## Boolean Algebra

- Two valued algebra
- Used to analyze the basic elements that digital computers are built from.
- A way of manipulating true/false values.

## Boolean Operations

- Basic operations:
  - AND – true iff both operands are true
  - OR – true if either or both operands are true
  - NOT – true when its operand is false (inverts the operand)
- Other common operations:
  - XOR – true if inputs are different
  - NAND – inversion of AND
  - NOR – inversion of OR
- 1 = true, 0 = false

## AND

- Truth table:

A	B	A AND B
0	0	
0	1	
1	0	
1	1	

A AND B can also be represented as:

$A \cdot B$   
 $AB$   
 $A \wedge B$

## OR

- Truth table:

A	B	A OR B
0	0	
0	1	
1	0	
1	1	

A OR B can also be represented as:

$A + B$   
 $A \vee B$

## NOT

- Truth table:

A	NOT A
0	
1	

NOT A can also be represented as:

$\hat{A}$   
 $A'$   
 $\neg A$   
 $A$

## XOR (Exclusive OR)

- Truth table:

A	B	A XOR B
0	0	
0	1	
1	0	
1	1	

A XOR B can also be represented as:

## 16 Possible Boolean Functions of Two Variables

- table from AoA, Chapter 2

## Identities of Boolean

### Algebra

AND form    OR form

Identity law	$1A = A$	$0 + A = A$
Null law	$0A = 0$	$1 + A = 1$
Idempotent law	$AA = A$	$A + A = A$
Inverse Law	$A\hat{A} = 0$	$A + \hat{A} = 1$
Commutative Law	$AB = BA$	$A + B = B + A$
Associative Law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive Law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
Absorption Law	$A(A + B) = A$	$A + AB = A$
De Morgan's Law		

- proof of AND form of distributive law using Truth Tables (on the board)

- proof of OR-form of DeMorgan's Law using truth tables (on board)

## Generating a Logic Function from a Truth Table

A	B	C	M
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- Find all the combinations that result in a one.
- Put them into an expression:

## Boolean Functions in Assembly

- Boolean functions fall into the category of bit-operations.
- What other bit operations have we seen?
- For boolean functions, the operations take place between the individual bits of the two operands.

## AND

- AND performs a bitwise AND operation between each bit of the two operands and places the result in the first operand.
- Formats:
  - AND reg, reg
  - AND reg, mem
  - AND reg, immed
  - AND mem, reg
  - AND mem, immed

## AND, cont.

- AND can clear selected bits in an operand while preserving (masking) the remaining bits.

```
mov al, 00111011b
and al, 00001111b ; al = 00001011b
```

The 00001111b is called a bit mask, it clears the upper four bits while preserving the lower four bits.

## AND Example

- Converting from lower case to upper case.

- Upper case letters have bit 5 set

```
.data
char db ?;put uppercase letter here
mask db 0DFh ; 11011111b
.code
mov ah, 1
int 21h ;get the char into AL
and al, mask ;mask out bit 5
mov char, al ;store uppercase char
```

## OR

- Performs a bitwise OR operation between each bit of the two operands and places the result in the first operand.
- Same formats as AND.
- OR is useful for setting certain bits to one while leaving the other bits unchanged.

```
mov al, 00111011b ;3Bh
or  al, 00001111b ;AL = 3Fh
the lower four bits of the result are set, the
others remain unchanged.
```

## OR Example

- Converting from upper case to lower.
- When we converted the other way, we cleared bit 5. Now we need to set it:

```
.data
char db '?;put lowercase letter here
setb db 20h ; 00100000b
.code
mov ah, 1
int 21h ;get the char into AL
or al, setb;set bit 5
mov char, al ;store lowercase char
```

## Another OR Example

;converting a decimal digit to ASCII

```
DIGIT DW 7
ASCBias DW 30h
....
MOV AX, DIGIT
OR AX, ASCBias
```

## Checking for Set Bits

- AND can be used to see if a bit is set in a word:  
; test if bit 2 of BX = 0. If yes, jump  
mov ax, bx ; save original bx  
and ax, 0004h ; zero out all but bit 2  
jz zbit ;if zero (bit 2 zero), jump
- You can also use the TEST instruction: it does an AND but doesn't load results (implied AND)  
; test using TEST  
TEST bx, 0004h ;BX not changed  
JZ zbit ;but flags are set

## NOT

- NOT reverses all the bits in an operand (takes the 1's complement).
- Formats:
  - NOT reg
  - NOT mem

```
mov al, 11110000b
not al          ;al = 00001111b
```

## NEG

- NEG reverses the sign of a number by converting it to its two's complement.
- Formats:
  - NEG reg
  - NEG mem

```
mov al, +127 ;AL = 01111111b
neg al       ;AL = 10000001b
```

## Overflow with NEG

- You can get overflow:

```
mov al, -128 ;AL = 10000000b
neg al       ;AL = 10000000b,
              ;OF = 1
```

## XOR

- Performs a bit-by-bit exclusive OR, puts the result in the first operand.

```
mov al, 10110100b
xor al, 10000110b ; al = 00110010b
```
- Commonly used to set a register to zero:

```
XOR AX, AX ;same effect as mov ax, 0
```